



Full-field Phase Imaging in practice and the impact of improved coherence

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Lund University

Outline

I. Introduction to full-field imaging in the Fresnel regime

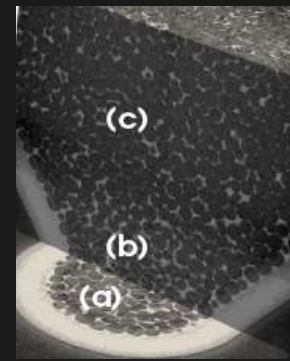
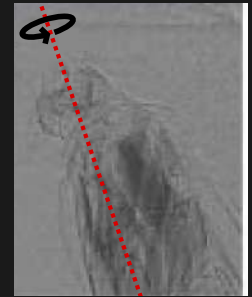
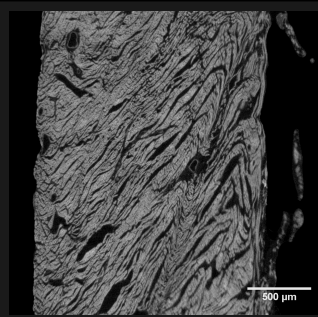
I. Overview of routines

II. Imaging at various scales of coherence

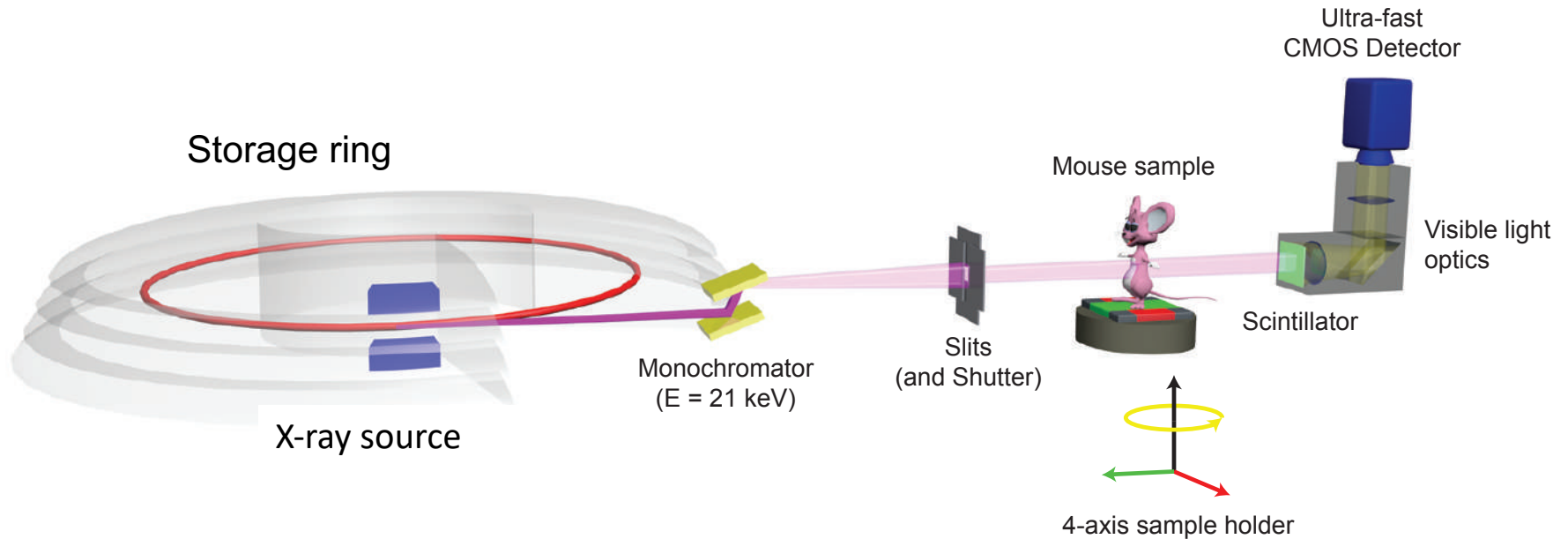
- I. Lab systems
- II. Synchrotron
- III. XFEL

III. Challenges

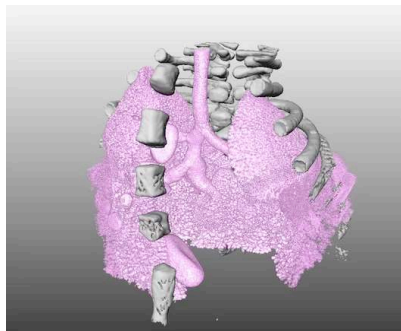
IV. Perspectives



Tomographic microscopy at synchrotron



Radiographic projection
clinics



Tomographic reconstruction
Synchrotron

Pixel size: 11 -> 0.3 μm

FOV: 22 x 22 -> 1 x 1 mm^2

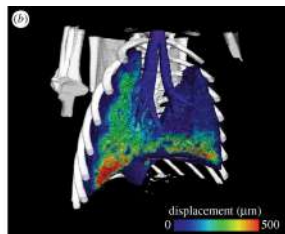
Projections: 300–2000

Exposure times: 0.1–300 ms

Total scan time: 0.05 to 500 s

Hierarchical imaging of biological samples

Pre-clinical imaging and therapy
(in vivo, medium resolution >30 μm , low dose, large beam, large FOV)



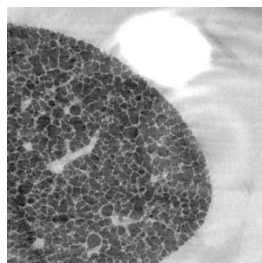
Longitudinal in vivo

The whole organ

2 mm

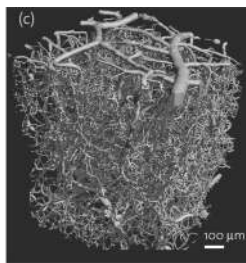


Down to a micrometer resolution
(rarely in vivo, high resolution $\sim 1\mu\text{m}$, higher dose, small FOV)



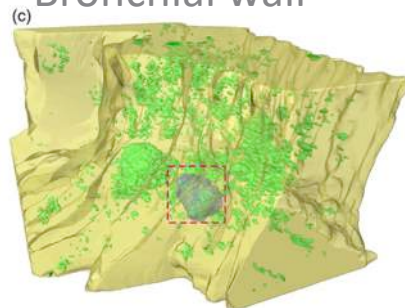
Acute in vivo

Zooming into organs



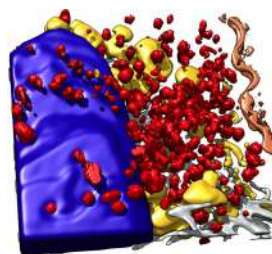
Nano-scale imaging beamlines
(fixed tissue, resolution $\sim 100\text{nm}$, high dose, small samples $\sim 100\mu\text{m}$)

Bronchial wall



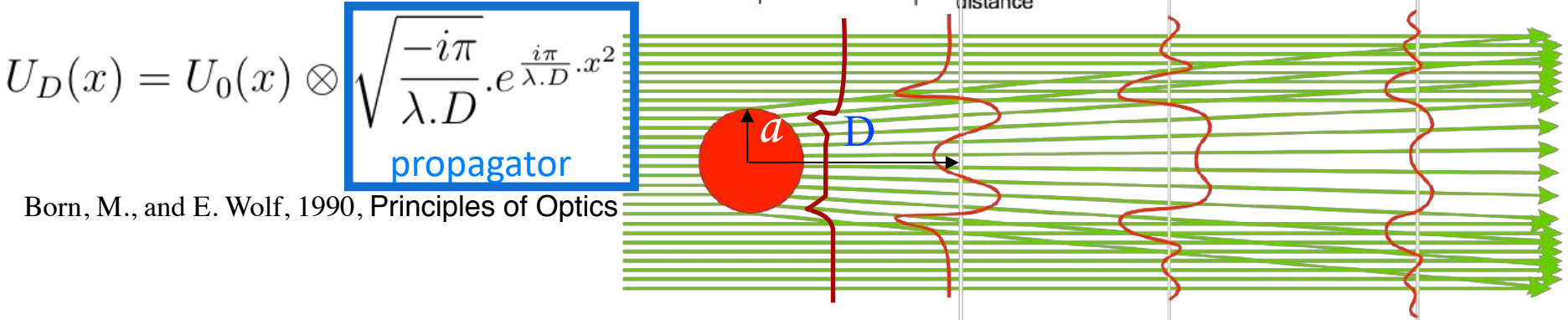
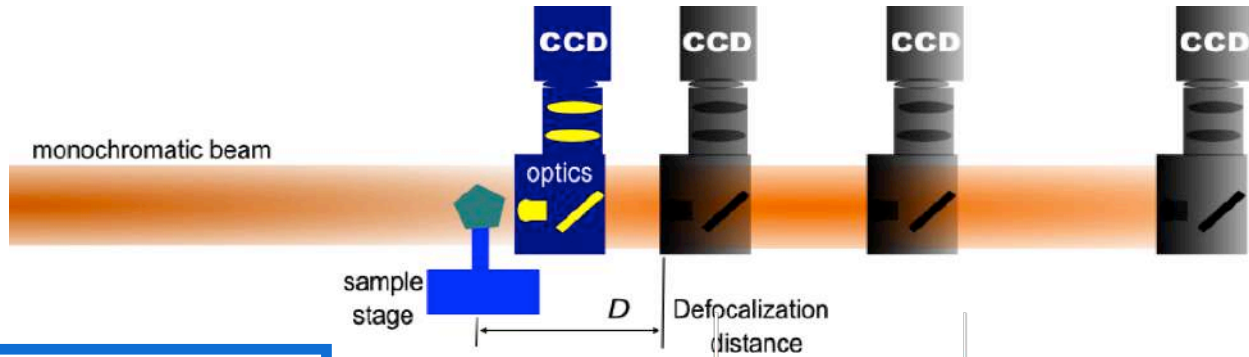
Fixed tissue

Cell level



3 orders of magnitude in spatial resolution

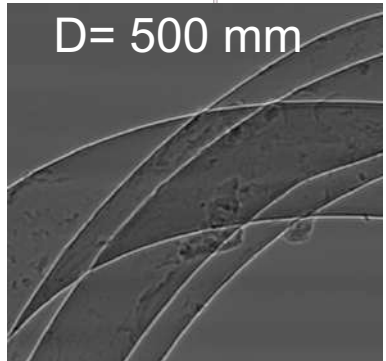
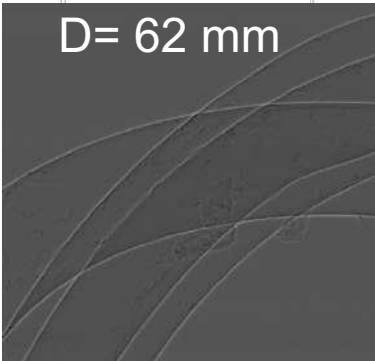
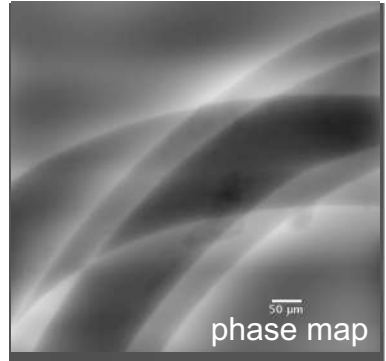
Phase contrast in free space propagation



$$U_D(x) = U_0(x) \otimes \sqrt{\frac{-i\pi}{\lambda \cdot D}} \cdot e^{\frac{i\pi}{\lambda \cdot D} \cdot x^2}$$

propagator

Born, M., and E. Wolf, 1990, Principles of Optics



Three phase fibers

Fresnel diffraction
Partially coherent illumination

$$\varphi(f) = \frac{\sum_m \sin(\pi \lambda D_m f^2) I_m(f)}{\sum_m 2 \sin^2(\pi \lambda D_m f^2)}$$

Cloetens et al. J. Phys. 1999

Free space phase imaging

Linear scenarios for near-field phase retrieval:

Transport of Intensity Equation (Small propagation distance)

$$-k \frac{I(\mathbf{x}; z) - I(\mathbf{x}; 0)}{z} = \nabla I(\mathbf{x}; 0) \nabla \psi(\mathbf{x}) + I(\mathbf{x}; 0) \Delta \psi(\mathbf{x})$$

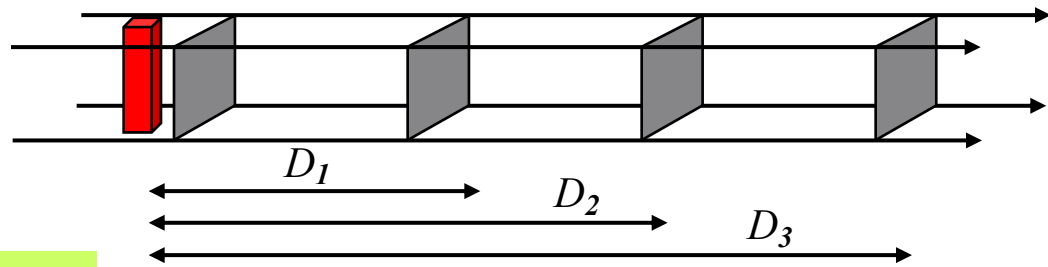
Teague, J. Opt. Soc. Am. 73, 1434 (1983)

Contrast Transfer Function (Weak scattering and absorption object)

$$I_D(\mathbf{f}) = \delta_{\text{Dirac}}(\mathbf{f}) - \underbrace{2 \cos(\pi \lambda D \mathbf{f}^2)}_{\text{amplitude contrast factor}} \cdot B(\mathbf{f}) + \underbrace{2 \sin(\pi \lambda D \mathbf{f}^2)}_{\text{phase contrast factor}} \cdot \varphi(\mathbf{f})$$

Guigay, Optik, 1977

Combination different distances
 D_m least squares minimisation



$$\tilde{\varphi}(f) = \frac{1}{2\Delta} \sum_{m=1}^N [A \sin(\beta_m) - B \cos(\beta_m)] \tilde{I}_m^{\text{exp}}$$

$$\tilde{B}(f) = \frac{1}{2\Delta} \sum_{m=1}^N [A \cos(\beta_m) - C \sin(\beta_m)] \tilde{I}_m^{\text{exp}}$$

direct method

$$A = \sum_m \sin(\beta_m) \cos(\beta_m)$$

$$B = \sum_m \sin^2(\beta_m)$$

$$C = \sum_m \cos^2(\beta_m)$$

$$\Delta = B \cdot C - A^2$$

$$\beta_m = \pi \lambda D_m f^2$$

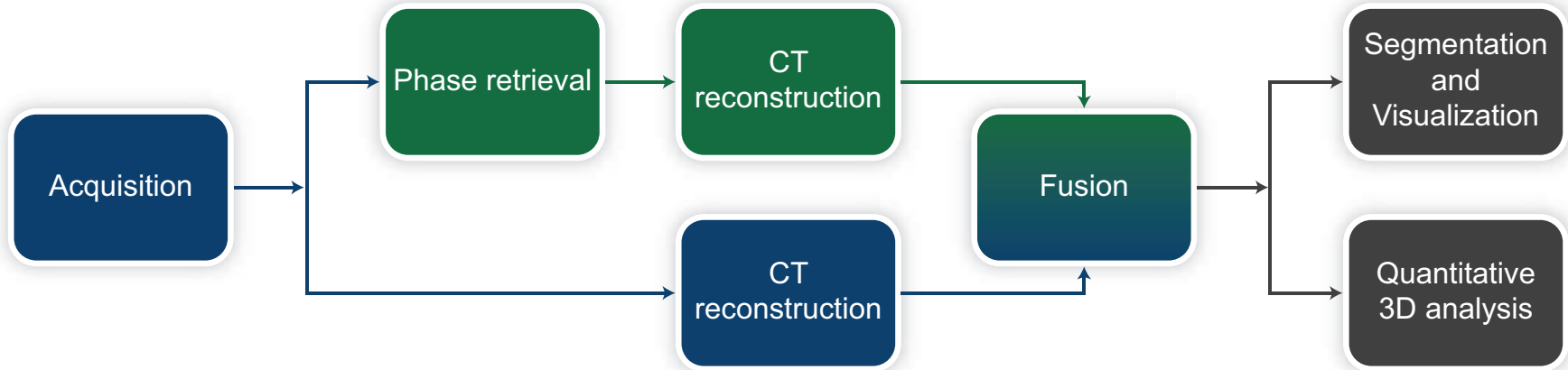
Cloetens, 1996 & 1999

Phase (contrast) tomography

Finding the phase of the object (real part of the refractive index)

$$I_D(\mathbf{x}) = \left| \text{FRT}_k[\mathbf{T}_{A,\phi}(\mathbf{x})] \right|^2$$

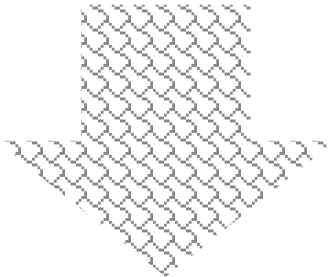
$$\phi(\mathbf{x}) = \arg \min_{\phi} \left\| \left| \text{FRT}_k[\mathbf{T}_{A,\phi}(\mathbf{x})] \right|^2 - I_D(\mathbf{x}) \right\|^2$$



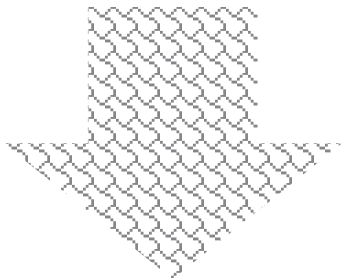
Motivation to utilize phase in full-field imaging

Contrast enhancement

- **Improve CNR** == less noisy images == reliably image interpretation / segmentation



- **reduce dose** enables high resolution in-situ and in vivo imaging



- **enhance acquisition speed** to avoid motion blur due to sample movement

Handling coherence in full-field imaging

How do we practically handle interference of coherent beams in full-field imaging?

c
o
m
p
l
e
x
i
t
y

- Retrieve the phase and attenuation deploying CTF or TIE based algorithms -> used mainly in nanoimaging with projection microscope
- Filter the projections with the Paganin filter to obtain the projected density -> very successful at beamlines because it is robust and simple
- Use the edge enhanced image -> often suitable for visual analysis
- We simply remove fringes to get clear attenuation image (e.g. decoherer on beamlines / Bronikov aided correction for lab sources)

Starting with low coherence

Lab microtomograph

4D IMAGING LAB@LTH



MAX IV

SoftiMAX

Soft X-rays
(in construction -> 2020)

NanoMAX

The nanofocus beamline
(users in 2017)

DanMAX

Imaging & diffraction
(in construction -> 2020)

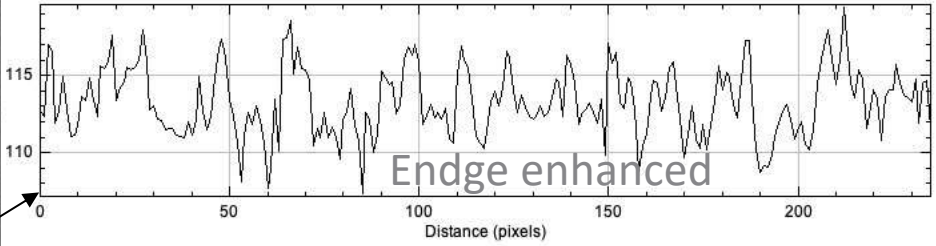
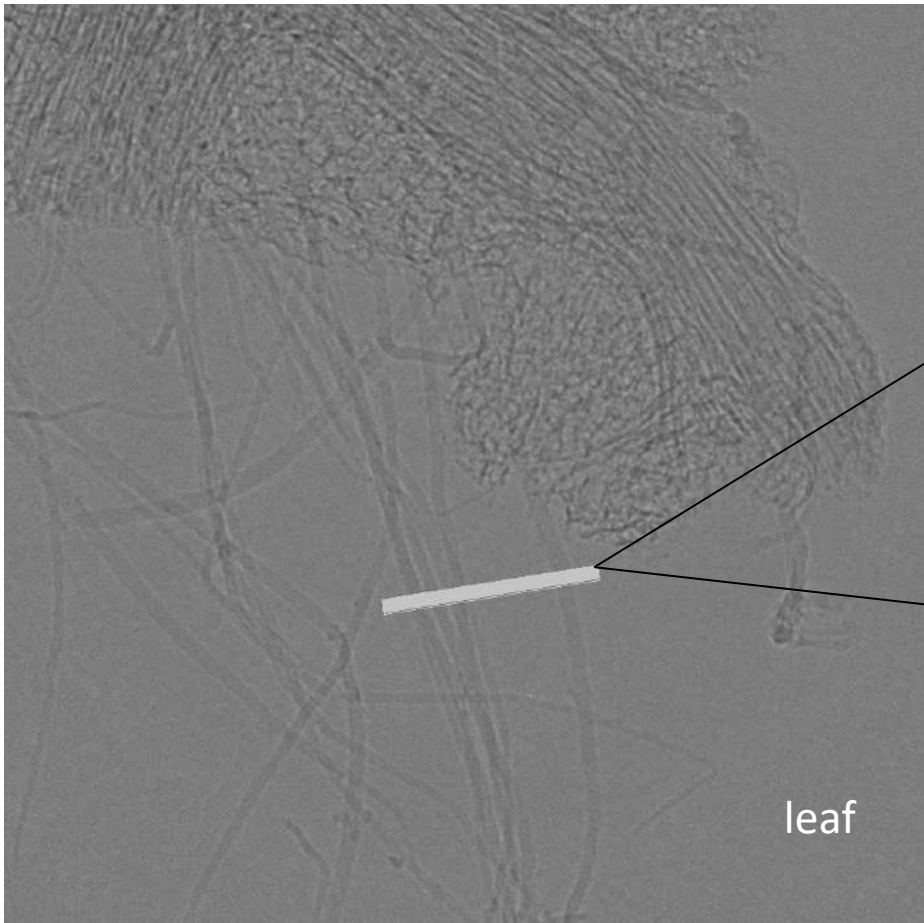


Giraffe heart

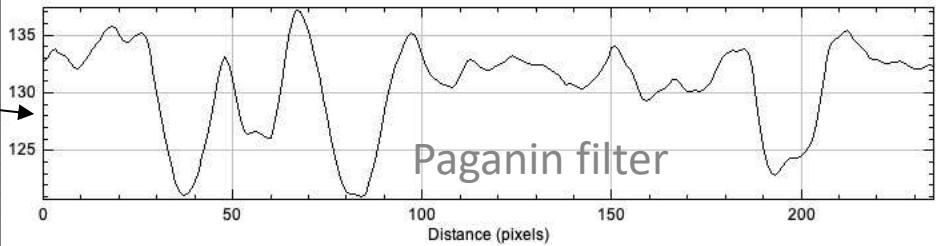
500 μm

Little coherence: lab sources

Edge enhancement vs. Paganin filtering for Versa 520



Profile plot



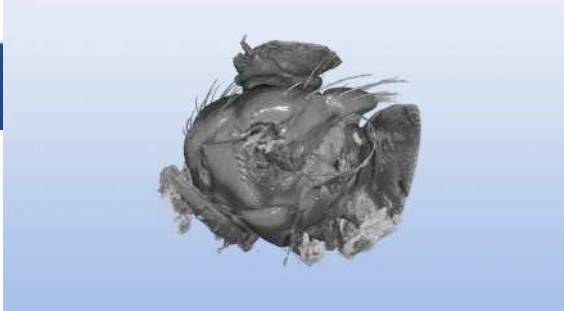
F. Mattsson, Master thesis, LTH

Exposure time = 1s, pixel size = 500 nm
60 kV (+filter) -> $E_{\text{effective}} \sim 30 \text{ keV}$

Medium coherence

Standard imaging beamline

Phase tomography



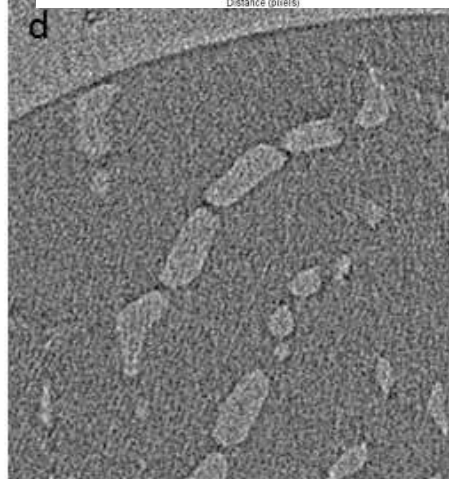
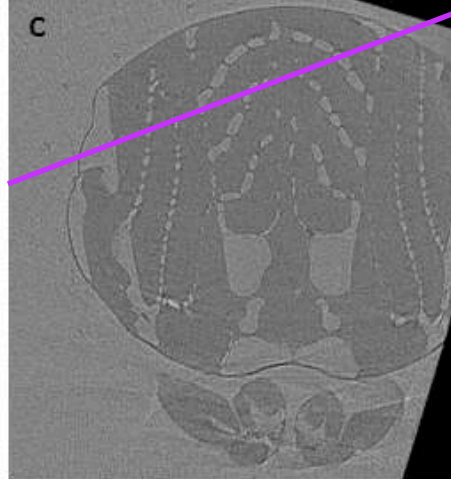
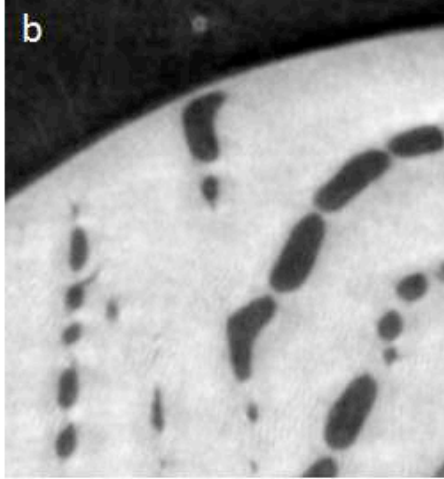
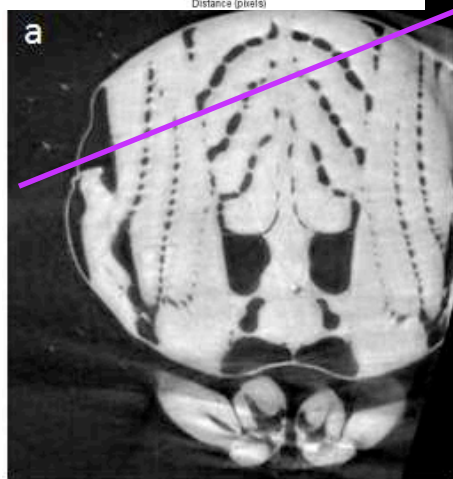
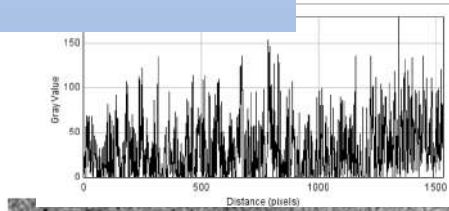
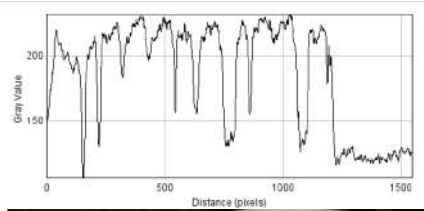
• Refractive index n

$$n = 1 - \frac{\delta}{\beta} + i \frac{\delta}{\beta}$$

$\delta \gg \beta$

fast tomography
30% of detector dynamic range

flight muscles of a fly



phase map

absorption & edge

Phase tomography

Phase contrast tomography

Image quality vs. radiation damage

2.9 μm pixel-size optics

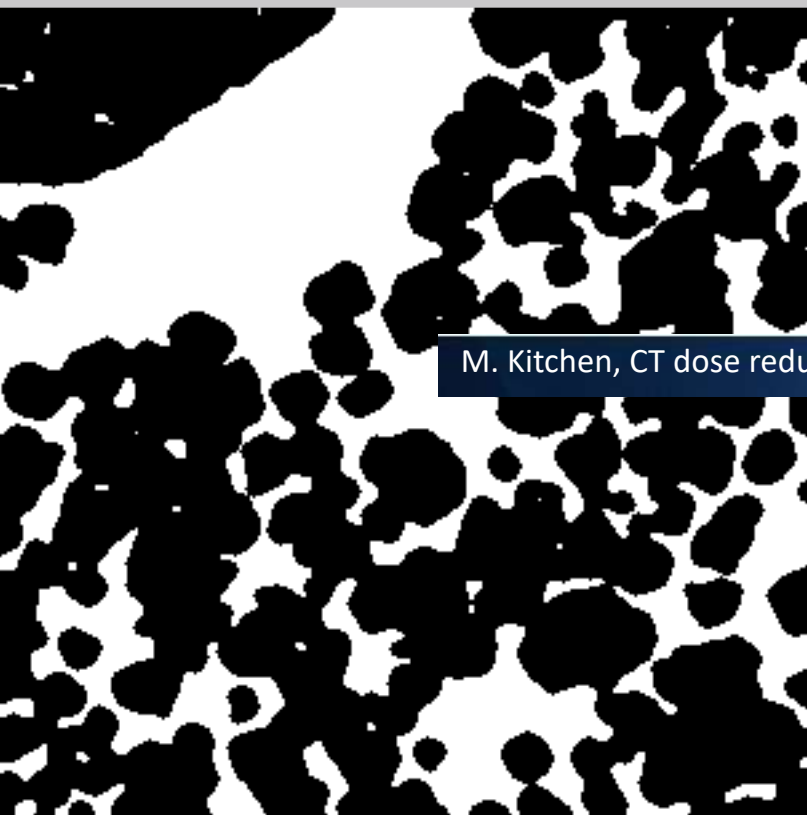
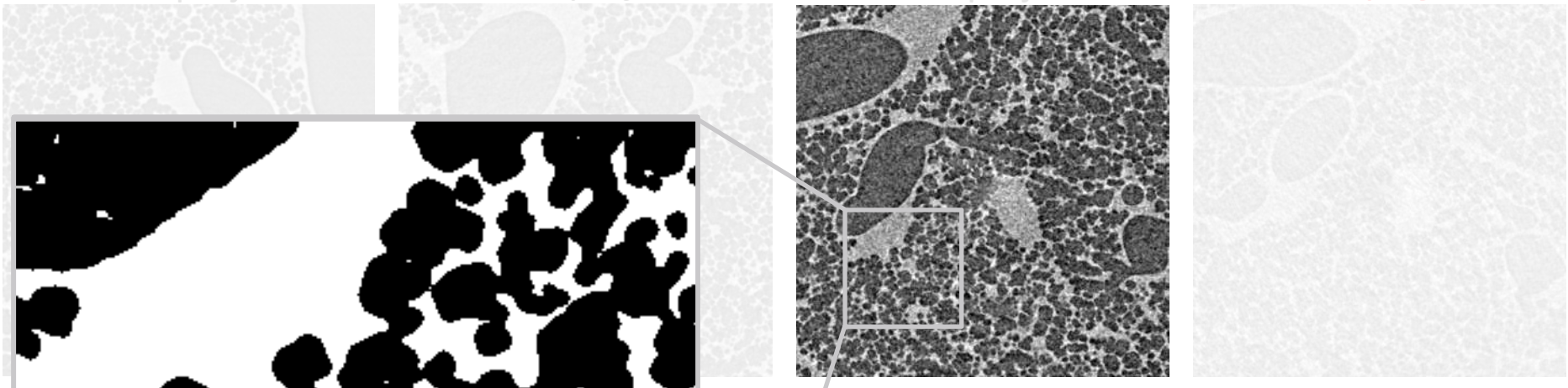
G. Lovric et al., J. Appl. Cryst. 46 (4) 2013

901 projections

901 projections

901 projections

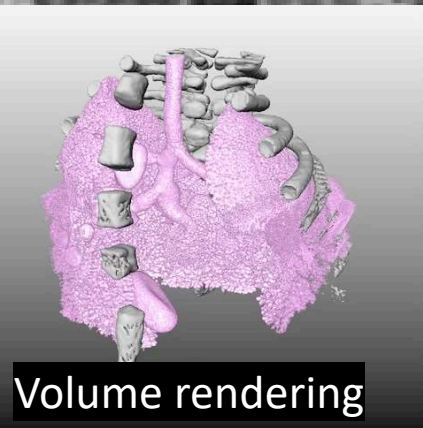
361 projections



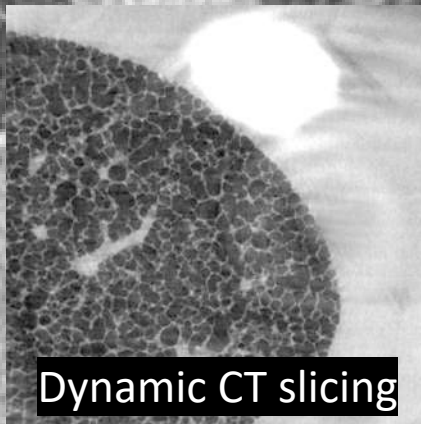
M. Kitchen, CT dose reduction factors in the thousands using X-ray phase contrast, Sci. Rep. 7 (2017)

0.24 s	0.17 s
CNR	CNR
2.3	1.5
Entrance dose	Entrance dose
12 Gy	9 Gy

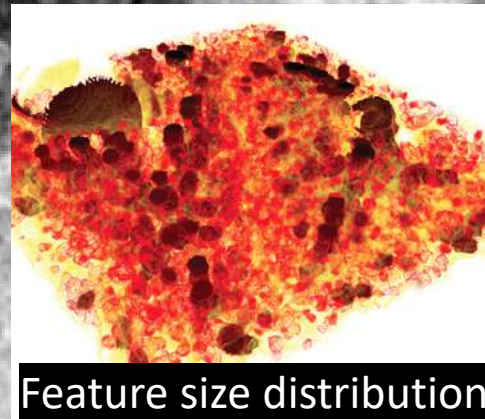
Lung alveoli microstructure in vivo



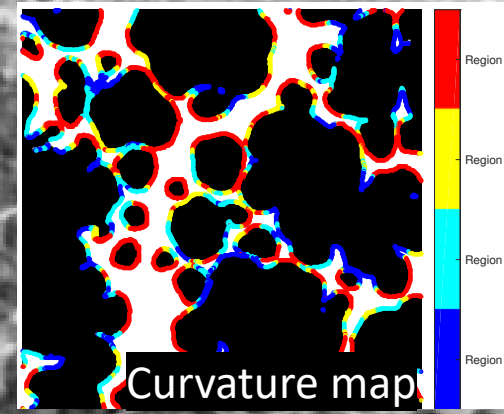
Volume rendering



Dynamic CT slicing



Feature size distribution



Curvature map

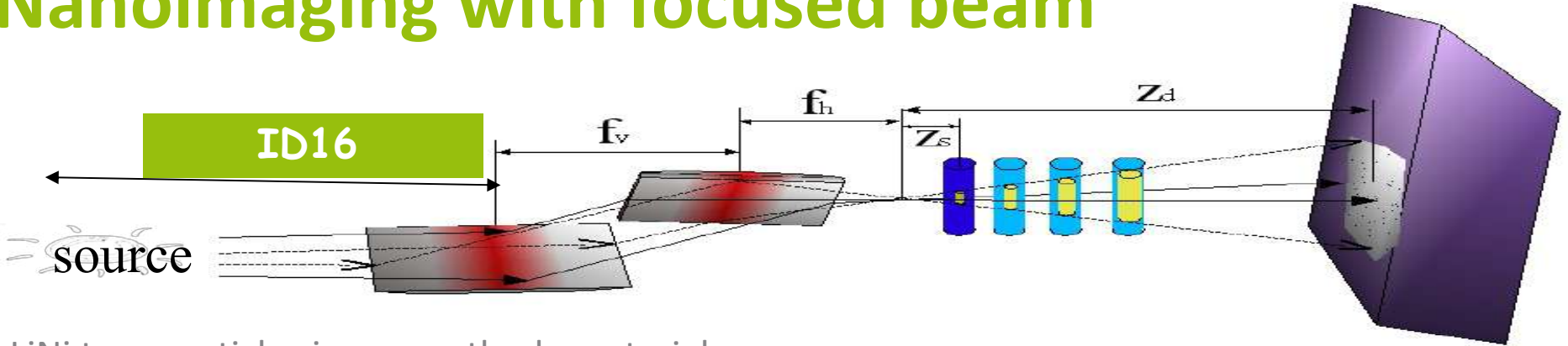
In vivo tomographic microscopy

Lovric et al. Sci. Rep. 2017

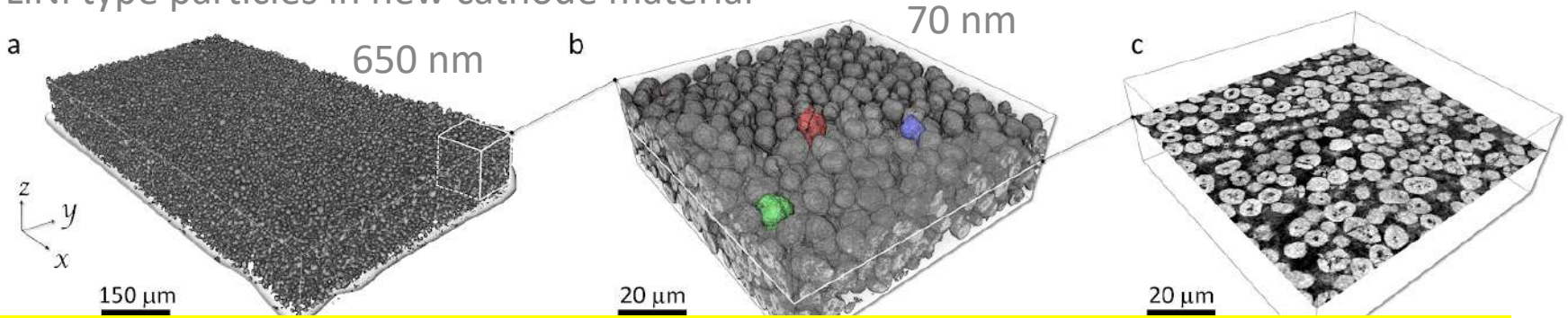
Highly coherent beams

Coherent imaging beamline

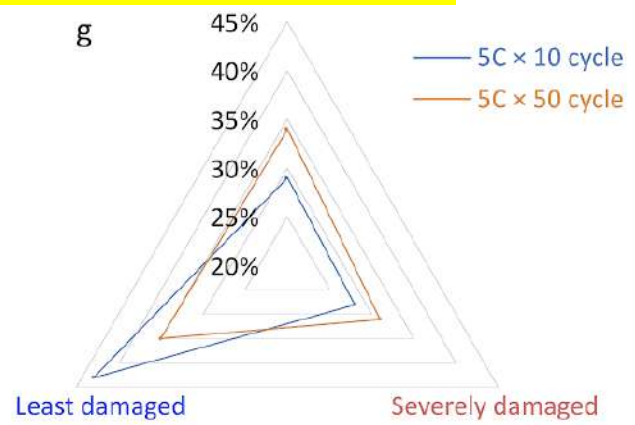
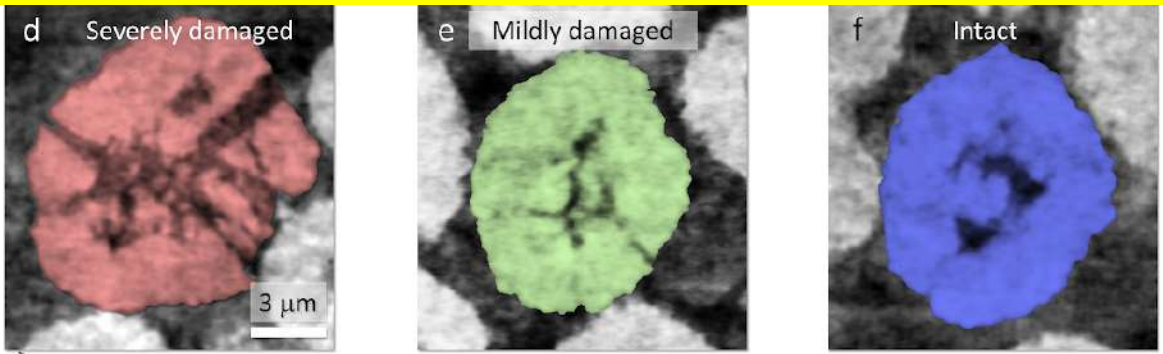
Nanoimaging with focused beam



LiNi type particles in new cathode material

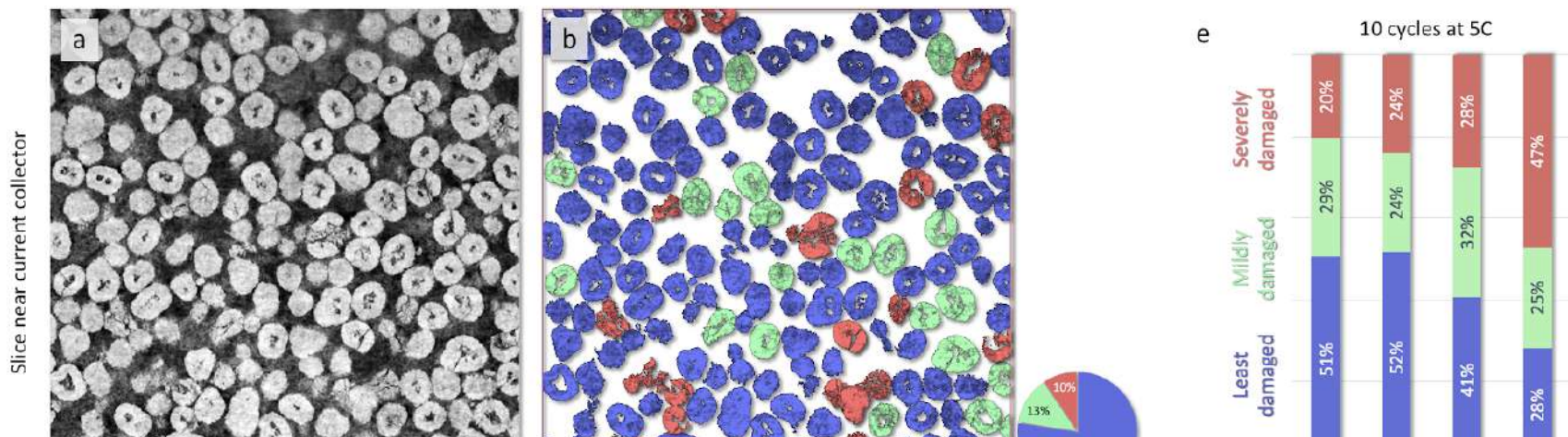


- Hierarchical morphology of secondary particles in Li-rich cathode materials.
- Develop at longer charging time 50cycle >10cycle

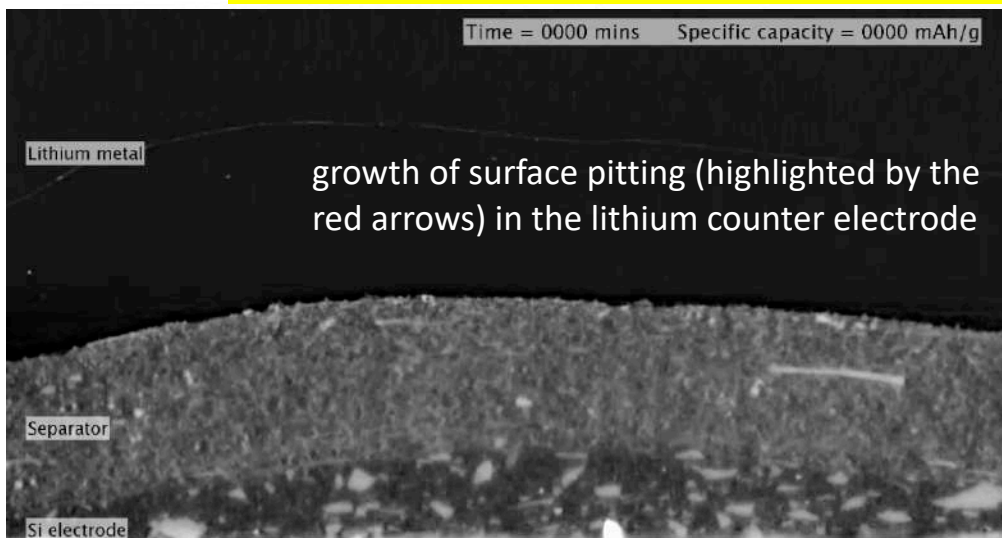


Y. Yang, Y. Liu, et al., Adv. Energy Mater. (2019)

Quantification of Heterogeneous Degradation in Li-ion Batteries

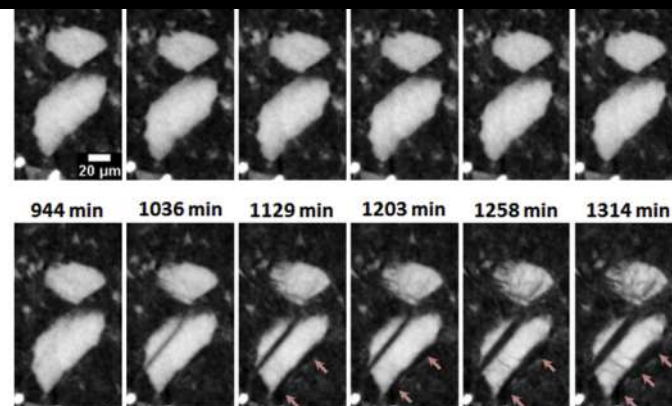


- A depth-dependent degree of particle fracturing.
- More damaged close to the separator than the current collector.

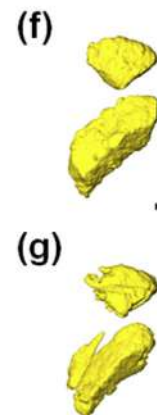


Y. Yang, Y. Liu, et al., *Adv. Energy Mater.* (2019)

In situ degradation of Si electrode / TOMCAT, SLS



Taiwo, et al., *J. Power Sources.* 342 (2017)



Development in phase retrieval algorithms

Working towards a more robust phasing tool

Direct model in phase retrieval, CTF

$$\tilde{I}_D(\mathbf{f}) = \delta(\mathbf{f}) - 2 \cos(\pi \lambda D |\mathbf{f}|^2) \tilde{B}(\mathbf{f}) + 2 \sin(\pi \lambda D |\mathbf{f}|^2) \tilde{\phi}(\mathbf{f}),$$

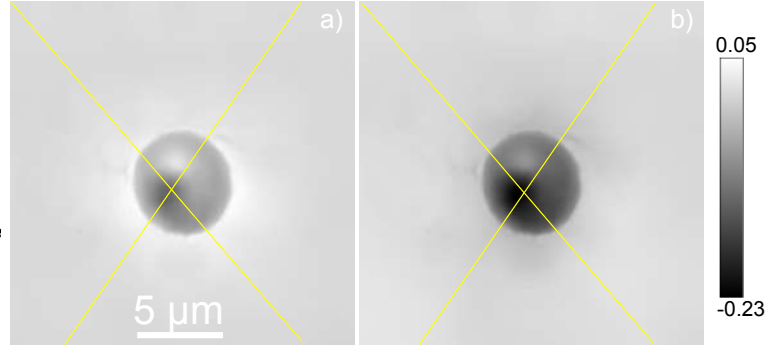


Fig. 6. Phase map of a red blood cell obtained through a) standard regularization and b) Bayesian inference.

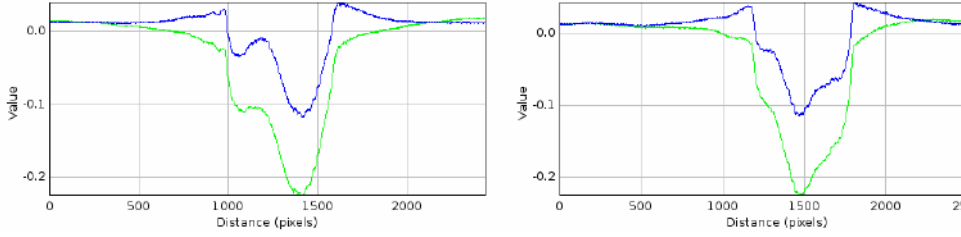
Solution by minimization of the Euklidian distance

$$\hat{\mathbf{x}} = \arg \min_{\mathbf{x}} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 = \arg \min_{\mathbf{x}} J_{LS}(\mathbf{x})$$

System matrix

Needs regularisation:

$$J(\mathbf{x}) = \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2 + \lambda_R \Delta(\mathbf{x}, \mathbf{x}_0).$$



Research Article Vol. 26, No. 25 | 10 Dec 2018 | OPTICS EXPRESS 32847

Optics EXPRESS

Robust determination of the regularisation parameter under Bayesian framework

Unsupervised solution for in-line holography phase retrieval using Bayesian inference

FLORIN FUS,^{1,2,*} YANG YANG,¹ ALEXANDRA PACUREANU,¹ SYLVAIN BOHIC,^{1,2} AND PETER CLOETENS¹



Phase retrieval with compressed sensing

- Phase problem:

$$\bar{\phi} = \min_{\phi} [|\mathbf{H}\phi - I|^2 + \eta R(\phi)], \text{ where } I = \mathbf{H}\phi;$$

- Contrast Transfer Function

Weak scattering absorption object

$$\hat{I}_z(\mathbf{f}_x) = 2\hat{\psi}(\mathbf{f}_x)\sin(\pi\lambda z|\mathbf{f}_x|^2) - 2\hat{B}(\mathbf{f}_x)\cos(\pi\lambda z|\mathbf{f}_x|^2)$$

$$\mathbf{H} = \mathcal{F}^{-1}(2\sin(\alpha|f|^2))\mathcal{F}$$

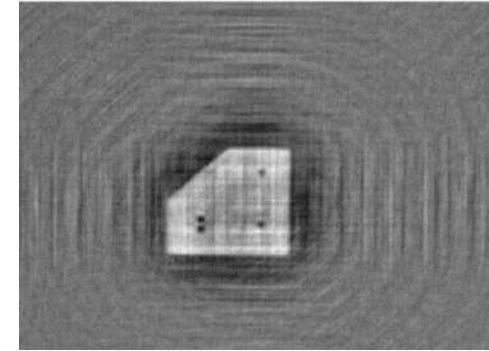
Neglecting attenuation

- ADMM lagrangian.

$$\mathcal{L}(\phi, u, \alpha) = \frac{1}{2} \|H\phi - I\|_2^2 + \lambda \sum_i \|u_i\|_2 - \alpha^T (u - \nabla\phi) + \frac{\beta}{2} \|u - \nabla\phi\|_2^2$$

- ADMM iteratively minimizes \mathcal{L} by solving smaller problems

b) Analytical reconstruction



(c) ADMM reconstruction



Letter

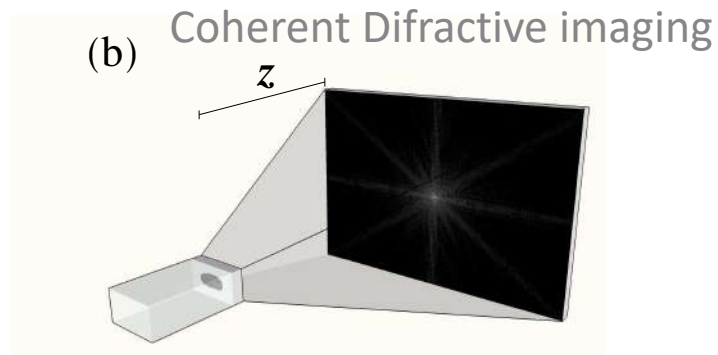
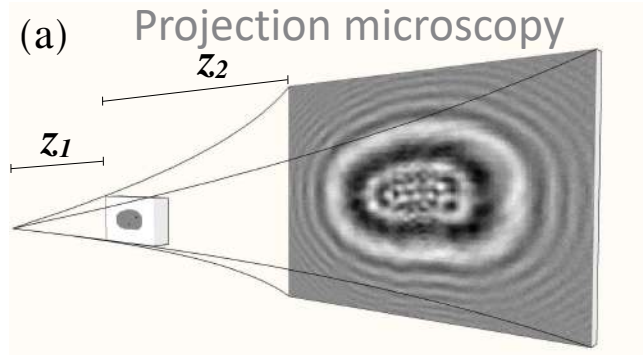
Vol. 42, No. 6 / March 15 2017 / Optics Letters

Optics Letters

Contrast-transfer-function phase retrieval based on compressed sensing

PABLO VILLANUEVA-PEREZ,^{1,4} FILIPPO ARCADU,^{1,2,5} PETER CLOETENS,³ AND MARCO STAMPANONI^{1,2}

Sensitivity, resolution and photon flux



Total number of scattered photons

$$N_S \equiv \Phi \sigma_S = \Phi \int_{-\infty}^{\infty} |1 - t(x, y)|^2 dx dy = \Phi \pi \sigma^2 |\phi_{\max}|^2$$

Coherent flux

Total scattering cross section

Gaussian feature width

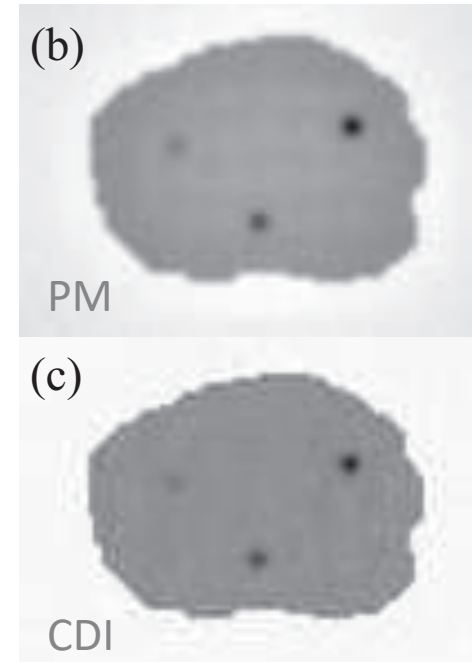
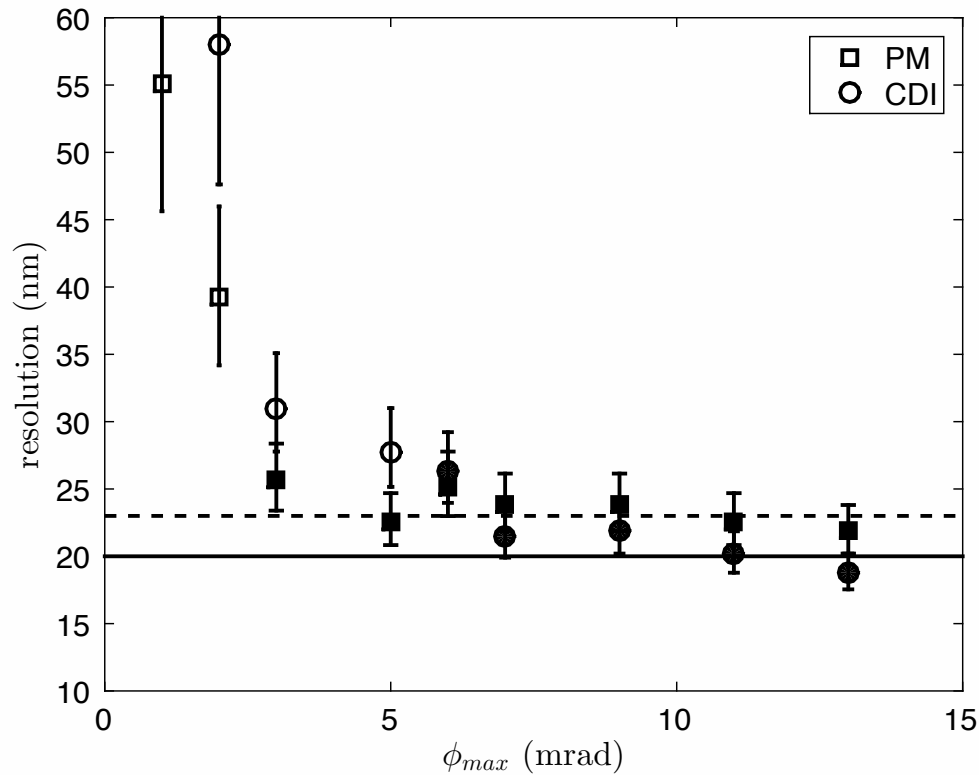
$$\text{SNR}_{\text{eff}}^{\text{PM}} \equiv 2\sqrt{N_S} \omega |A| \simeq 2\sqrt{N_S} \frac{4}{\sqrt{\pi}} \beta \frac{2\sigma}{\text{FOV}_{\text{PM}}}$$

$$\begin{aligned} \text{SNR}_{\text{eff}}^{\text{CDI}} &\equiv \sqrt{N_S} \frac{2\sqrt{\pi}\sigma}{\text{FOV}_{\text{CDI}}} \frac{2\pi}{\pi q_{\max}^2} \int_0^{q_{\max}} e^{-\frac{\sigma^2 q^2}{2}} q dq \\ &\approx \sqrt{N_S} \frac{2\sigma}{\text{FOV}_{\text{CDI}}} \end{aligned}$$

Villanueva et al. Opt. Exp. 2016

Sensitivity, resolution and photon flux

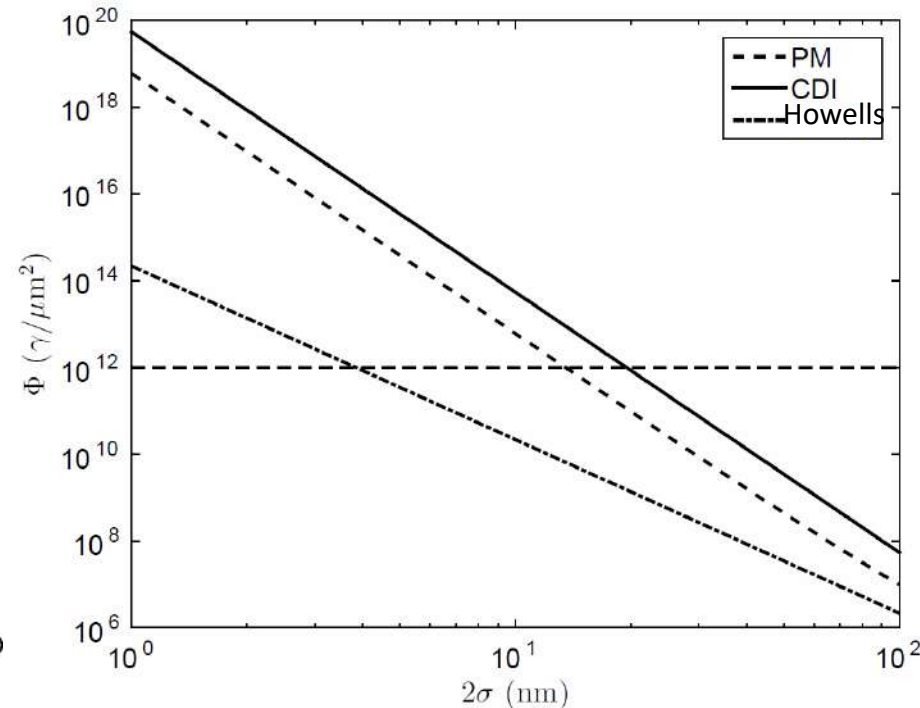
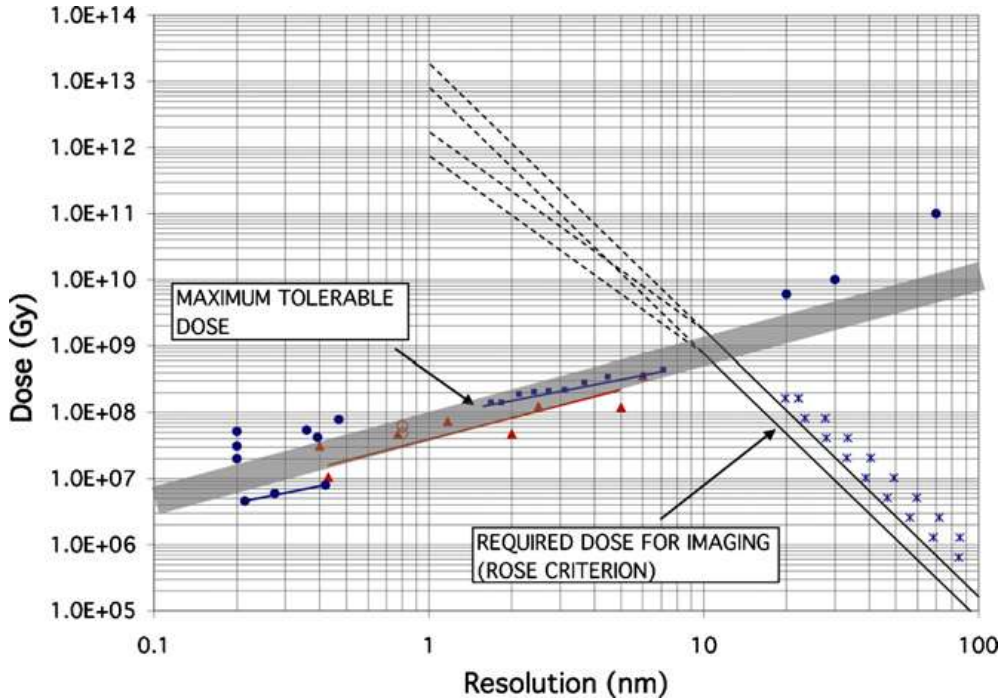
Resolution as a function of the maximum phase shift of the sample.



Sensitivity, resolution and photon flux

The number of required photons is proportional to the desired spatial resolution.

Required imaging fluence as a function of feature size (2σ)



Howels et al., J. El.Spectr.2009

Villanueva et al. Opt. Exp. 2016

Total entrance dose

$$D[\text{Gy}] = \phi \left(\frac{\mu}{\rho} \right) \cdot h\nu \cdot t_{\text{scan}}$$

ϕ ... flux [photons/s]

$h\nu$... energy per photon

t_{scan} ... total scan time

Full-field imaging demo from MAX IV

See poster

NanoMAX experiment

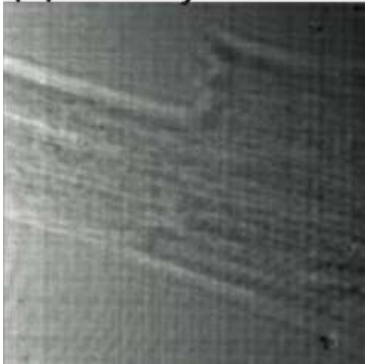
Focus characterisation of the NanoMax Kirkpatrick-Baez mirror system



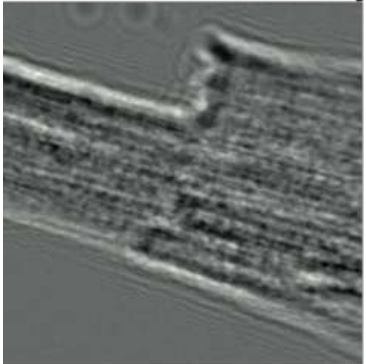
Markus Osterhoff¹, A.L. Robisch¹, S. Kalbfleisch², J. Soltau¹, M. Eckermann¹,
D. Carbone², U. Johansson², T. Salditt¹

¹Institut für Röntgenphysik - Friedrich-Hund-Platz 1 - 37077 Göttingen, Germany
²MAX IV Laboratory - Lund University - Fotogatan 2 - 22484 Lund, Sweden

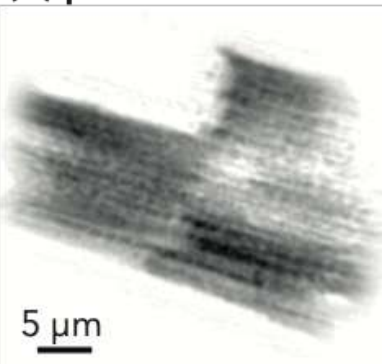
(b) intensity



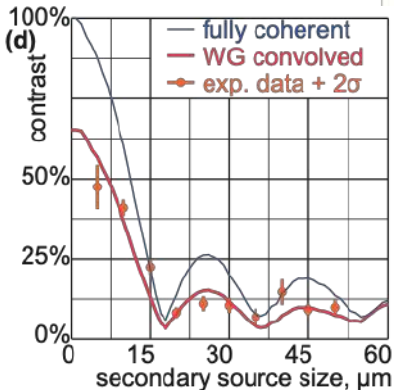
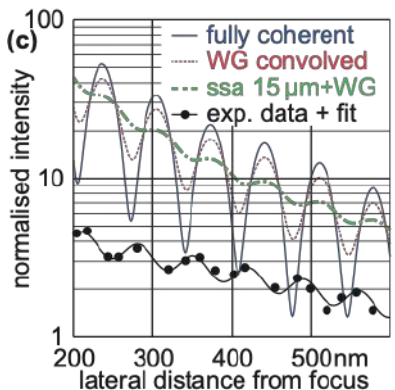
(c) normalised intensity



(d) phase



D.R. Luke: Relaxed averaged alternating reflections for diffraction imaging. Inverse Problems 21, 37 (2005)



(Fully) coherent beams

European XFEL

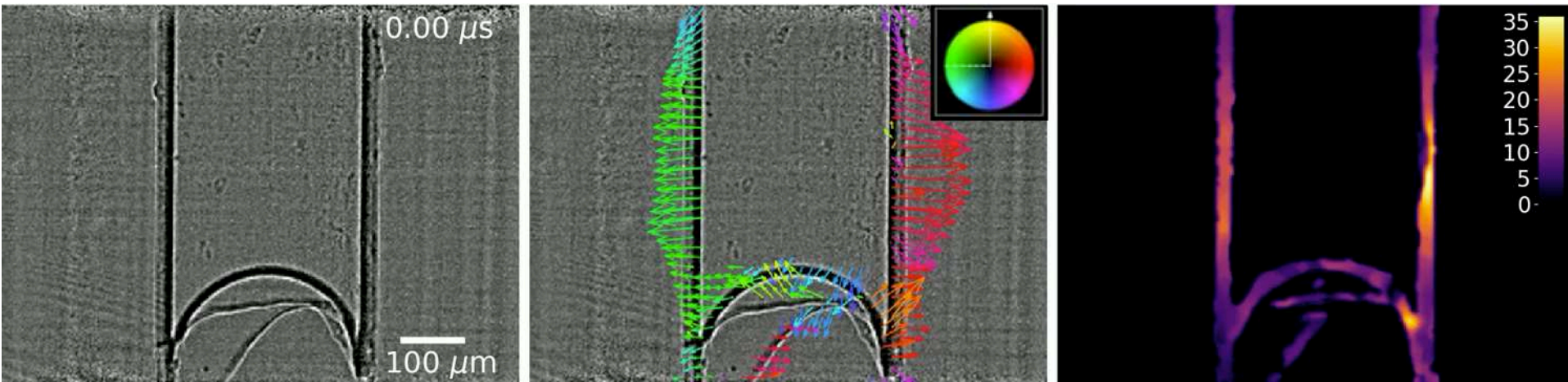
Highest coherence

optica

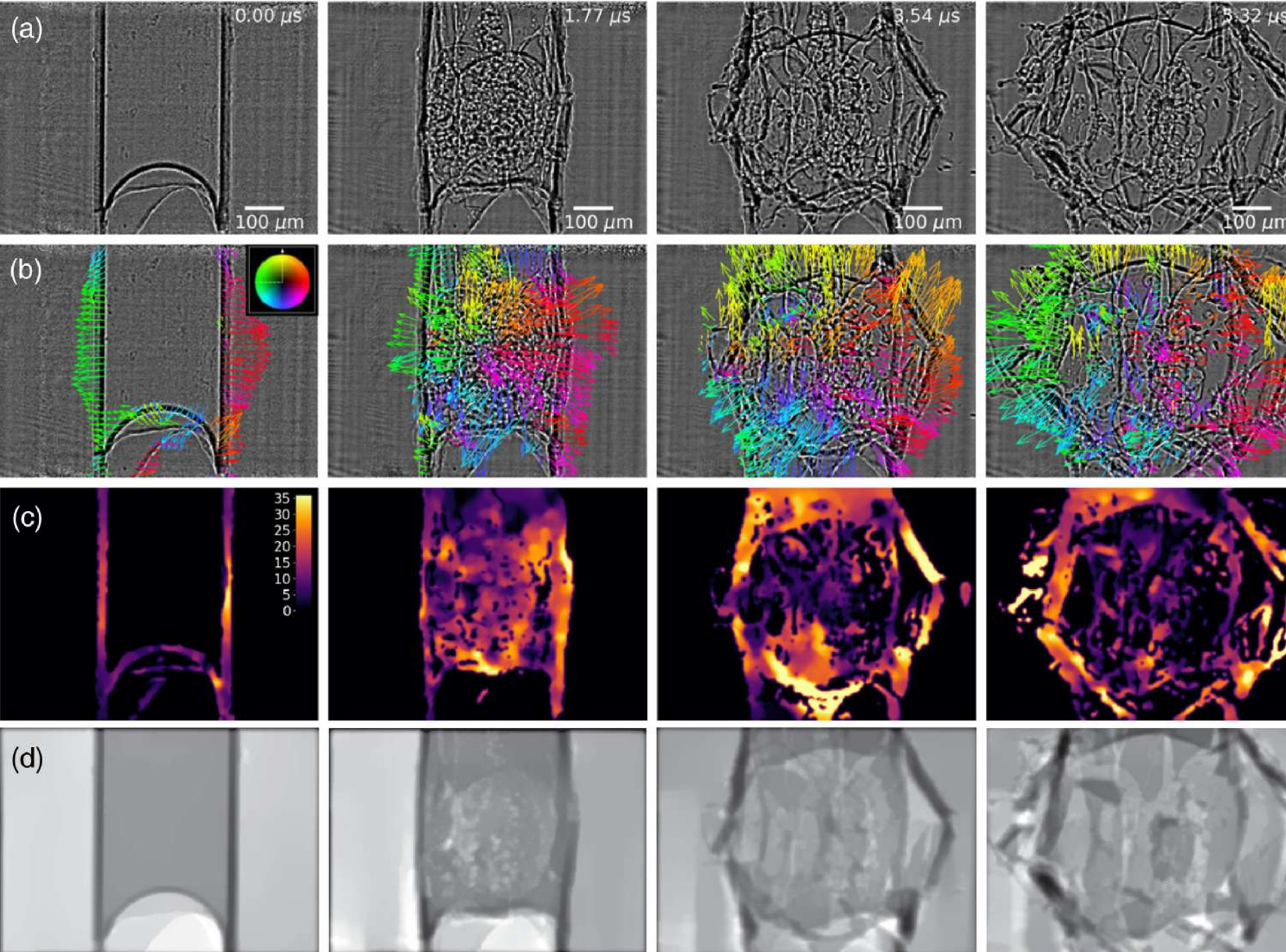
Megahertz x-ray microscopy at x-ray free-electron laser and synchrotron sources

PATRIK VAGOVIČ,^{1,2,3,*} TOKUSHI SATO,^{1,2} LADISLAV MIKEŠ,² GRANT MILLS,² RITA GRACEFFA,² FRANS MATTSSON,⁴ PABLO VILLANUEVA-PEREZ,^{4,1} ALEXEY ERSHOV,⁵ TOMÁŠ FARAGÓ,⁵ JOZEF ULIČNÝ,⁶ HENRY KIRKWOOD,² ROMAIN LETRUN,² RAJMUND MOKSO,⁴ MARIE-CHRISTINE ZDORA,^{7,8,9} MARGIE P. OLBINADO,¹⁰ ALEXANDER RACK,¹⁰ TILO BAUMBACH,⁵ JOACHIM SCHULZ,² ALKE MEENTS,¹ HENRY N. CHAPMAN,¹ AND ADRIAN P. MANCUSO^{2,11}

MHz radiography



$E = 9.3$ keV,
 the pulse train was filled with 128 x-ray pulses with a repetition rate of 1.128 MHz.
 The effective pixel size of the imaging system was $3.2 \mu\text{m}$

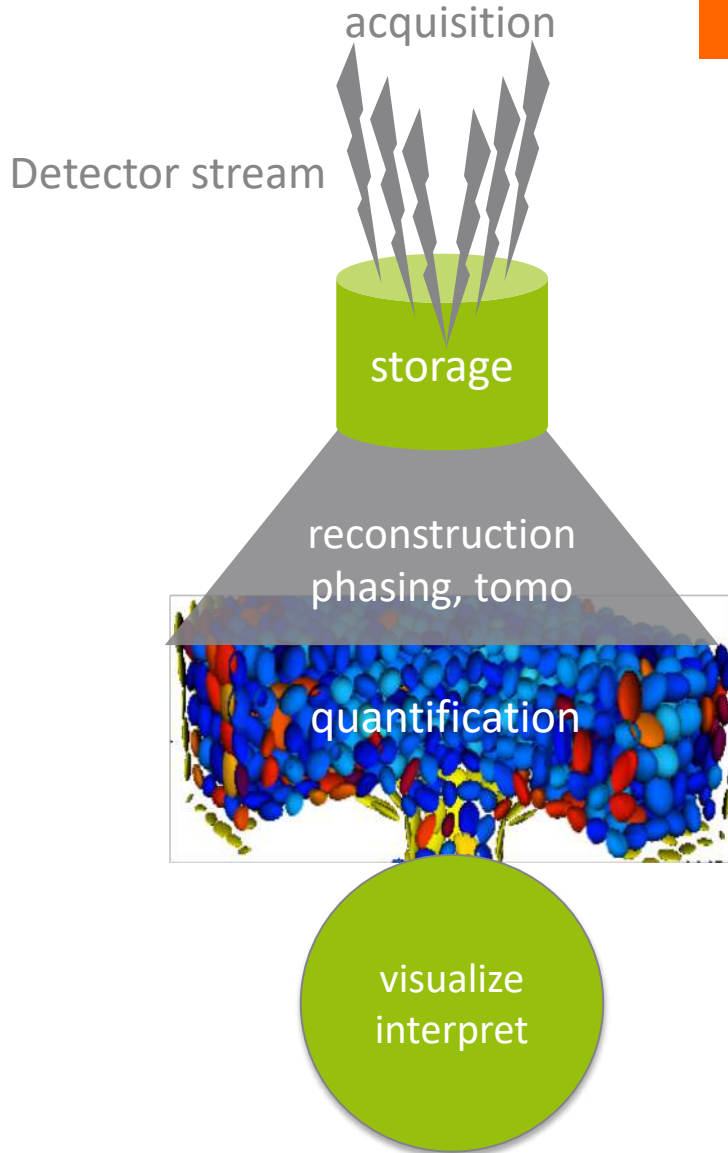


Challenges

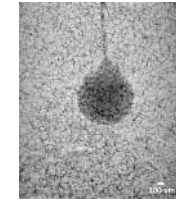
Image reconstruction and analysis remains the bottleneck

Turning data into science

1TB / 2 min

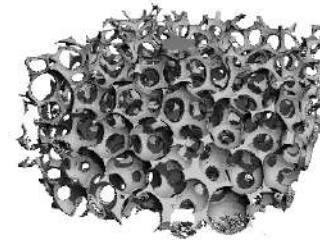


300 GB



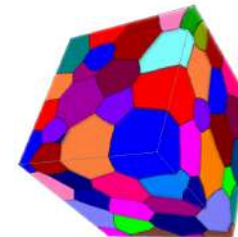
projections

2 x 300 GB



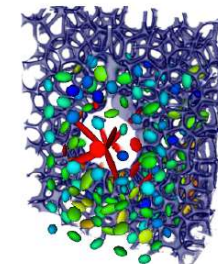
tomograms

1 GB



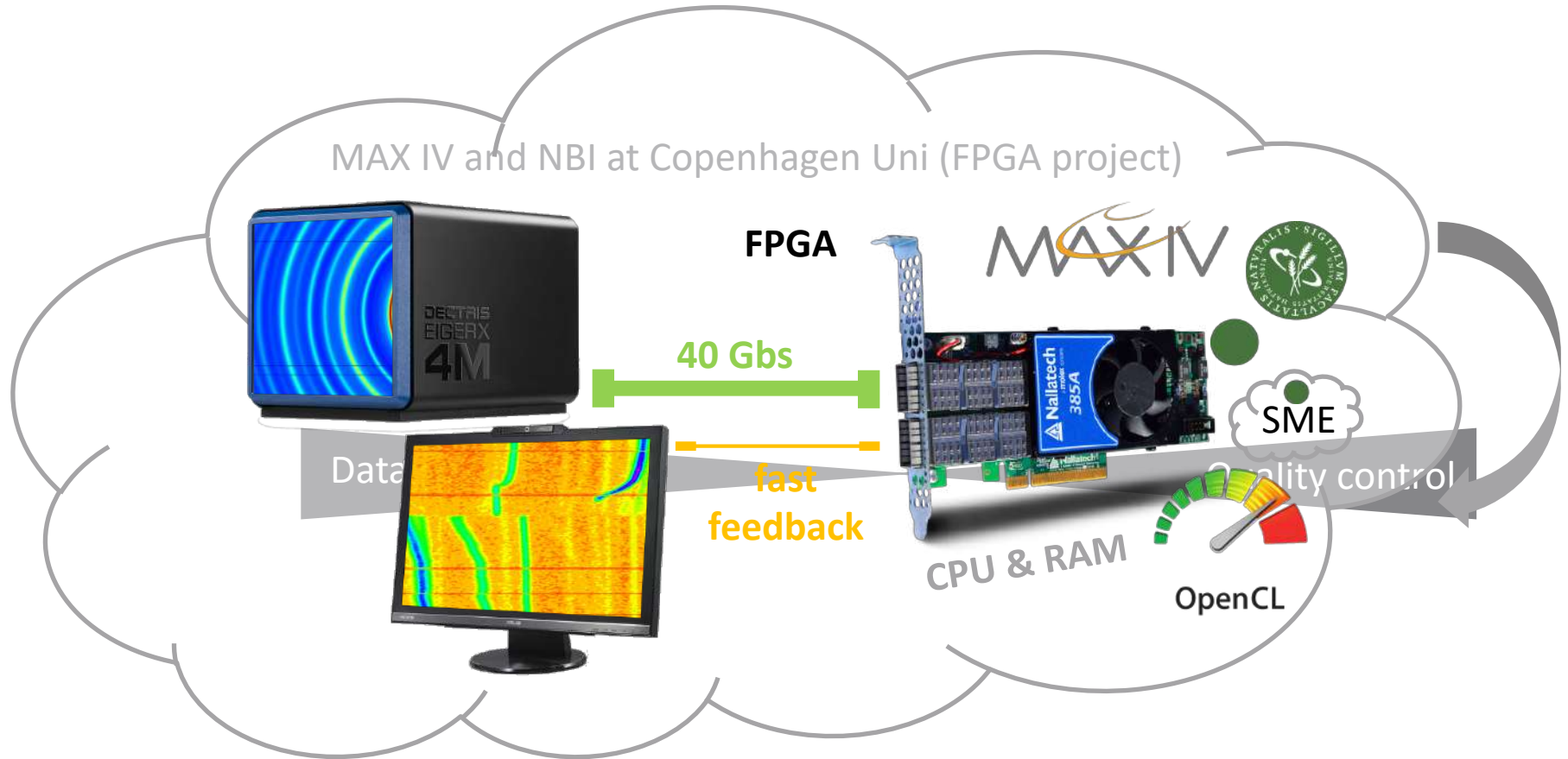
features

1 MB



shapes

Processing data streams for quality check and reduction



Data reduction in tomo: minimize number of scans saved per experiment.

E.g. foam dynamics studies contain lot of static scans. One could correlate consecutive 3D volumes and if they match one is deleted.

Dynamic tomographic reconstruction

Time domain decomposition

$$\mathcal{R}_\alpha f(s, z, \theta) = \iiint f(x, y, z, t) \delta(x \cos \theta + y \sin \theta - s) \delta(\theta - \alpha t) dx dy dt$$

Decomposition

$$f(x, y, z, t) \approx \sum_{j=0}^{M-1} f_j(x, y, z) \varphi_j(t),$$

where $\{f_j(x, y, z)\}_{j=0}^{M-1}$ are decomposition coefficients.

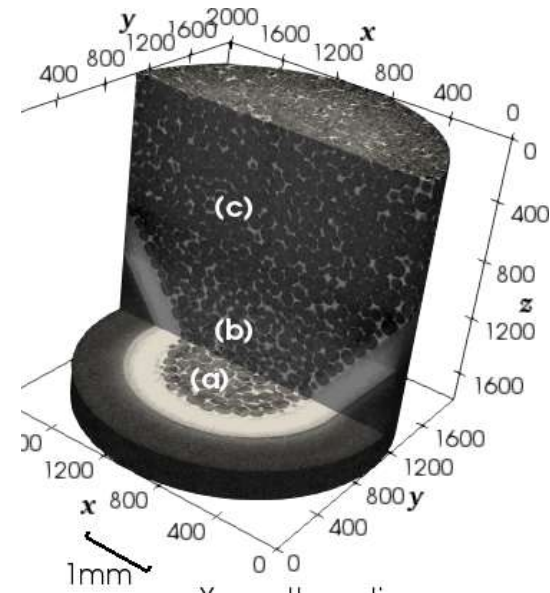
Example: Fourier basis $\varphi_j(t) = e^{2\pi i t \xi_j}$.

By using the linearity property of the projection operator:

$$\mathcal{R}_\alpha f(s, z, \theta) = \sum_{j=0}^{M-1} \mathcal{R} f_j(\theta, s, z) \varphi_j \left(\frac{\theta}{\alpha} \right)$$

$$\mathcal{R}_\alpha^* g(x, y, z, t) = \sum_{j=0}^{M-1} \varphi_j(t) \mathcal{R}^* (g \hat{\varphi}_j) (x, y, z),$$

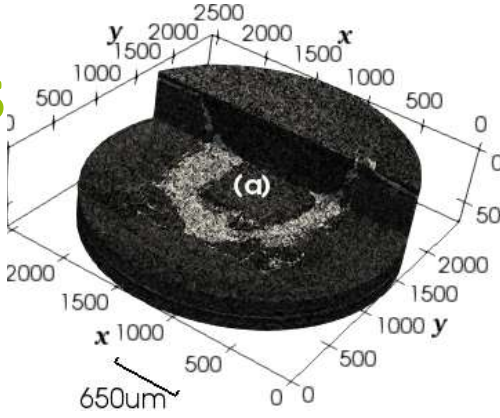
where $\mathcal{R}, \mathcal{R}^*$ are standard projection and back-projection operators in the static case.



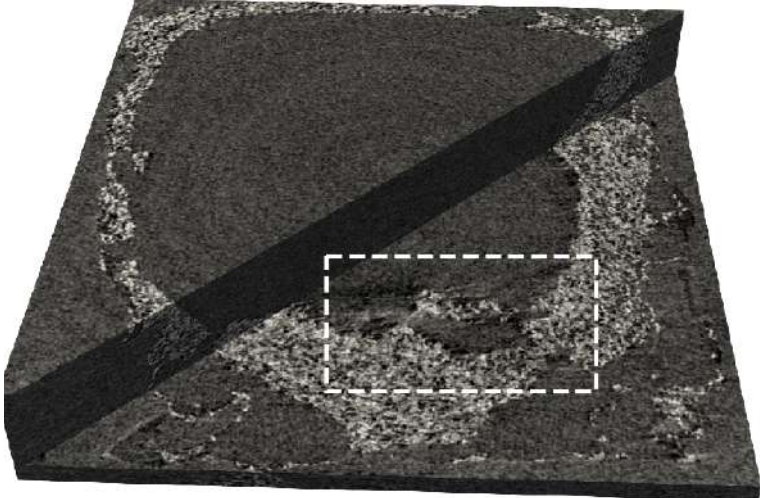
2016 x 2016 x 1800
130 rotations

Dynamic tomography: ceramic particles

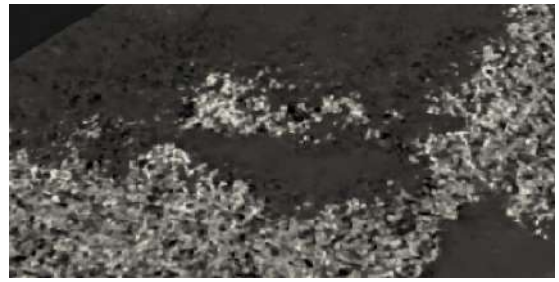
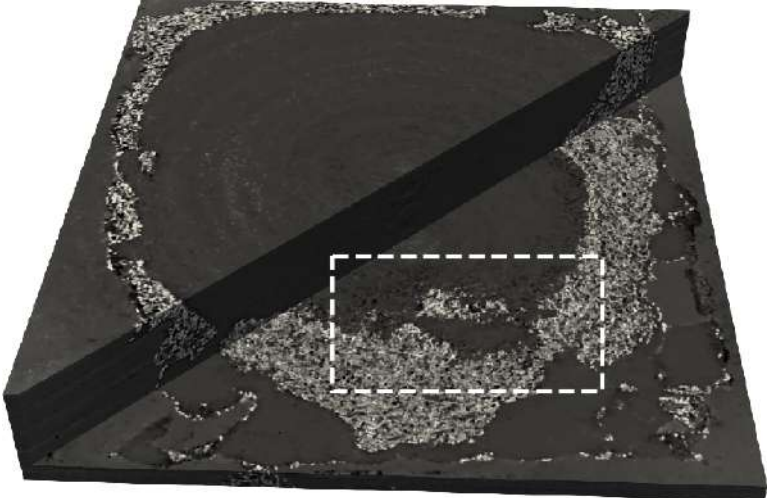
Tomography at 2BM, APS
2560 x 2560 x 700, 12 x 900 projections



FBP reconstruction



Iterative, basis size M=24



... and here is how to use the new tool

Foam data <https://tomobank.readthedocs.io>

In this study, we investigate the rheology of liquid foams by fast synchrotron X-ray tomographic microscopy [B30]. Foams are complex cellular systems which require artifact free tomographic reconstruction for a reliable quantification of their time-dependent properties such as deformation fields of bubbles. In our example we acquire X-ray projections of the liquid foam flowing through a constriction and being rotated around the tomographic axis. The experiment was performed at the TOMCAT beamline of the Swiss Light Source using the fast acquisition setup [B28].

To load the data sets and perform reconstruction use the [tomopy_rectv.py](#) python script.

Reconstruction by Gridrec

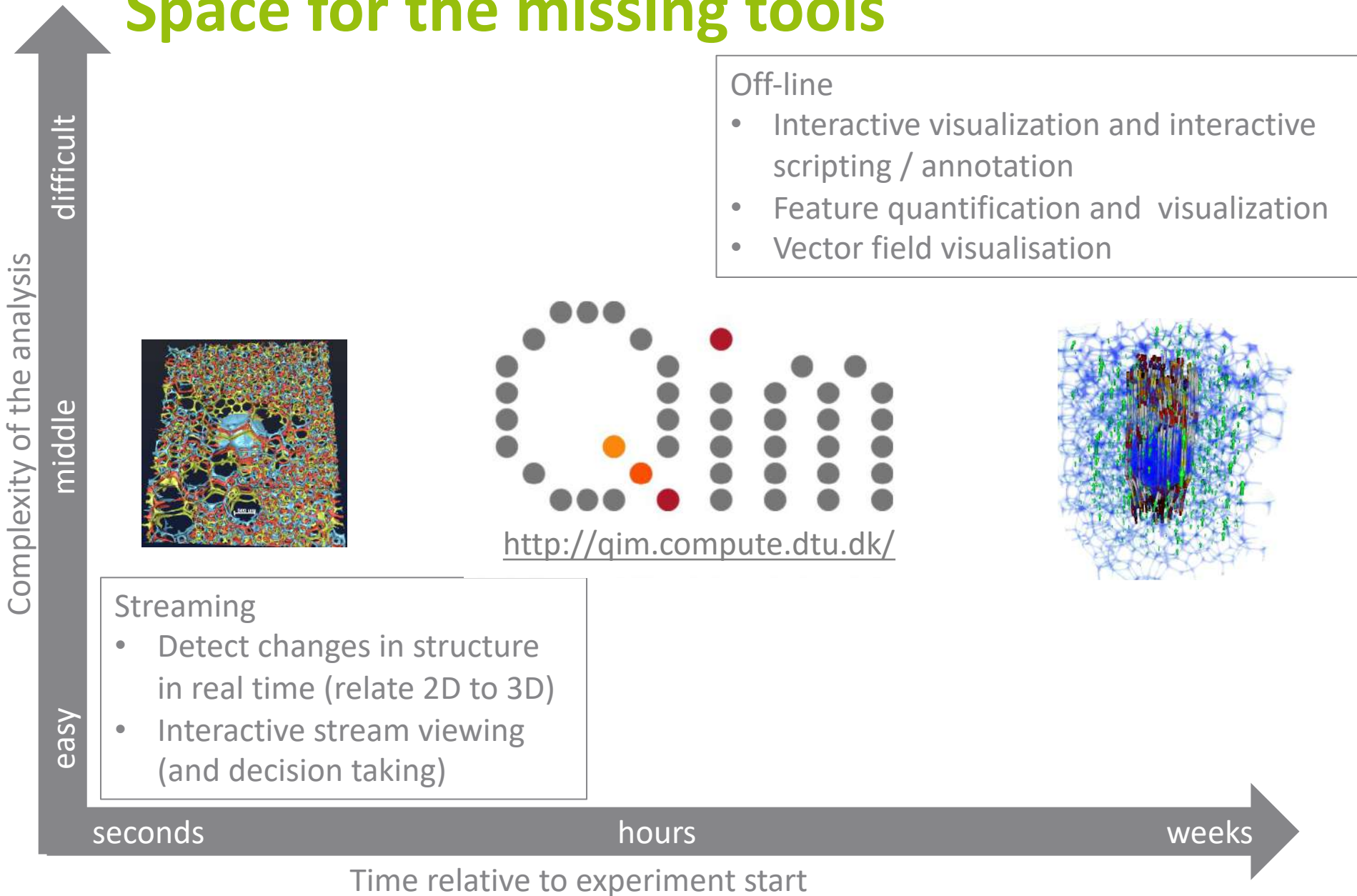
```
python tomopy_rectv.py dk_MCFG_1_p_s1_h5 --type subset --nsino 0.75 --binning 2 --frame 95
```

Reconstruction by the method with suppressing motion artifacts [Nikitin et. al, 2018] requires module *rectv* that can be installed from https://github.com/math-vrn/rectv_gpu. In this case, the algorithm run with option `-tv True`

```
python tomopy_rectv.py dk_MCFG_1_p_s1_h5 --type subset --nsino 0.75 --binning 2 --tv True --frame
```

tomo_ID	00080
Image preview	
Downloads	tomo_00080
Instrument	SLS TOMCAT
Sample name	dk_MCFG_1_p_s1
Energy	16 keV
Sample-to-detector Distance	250 mm
Scan Range	180 degree

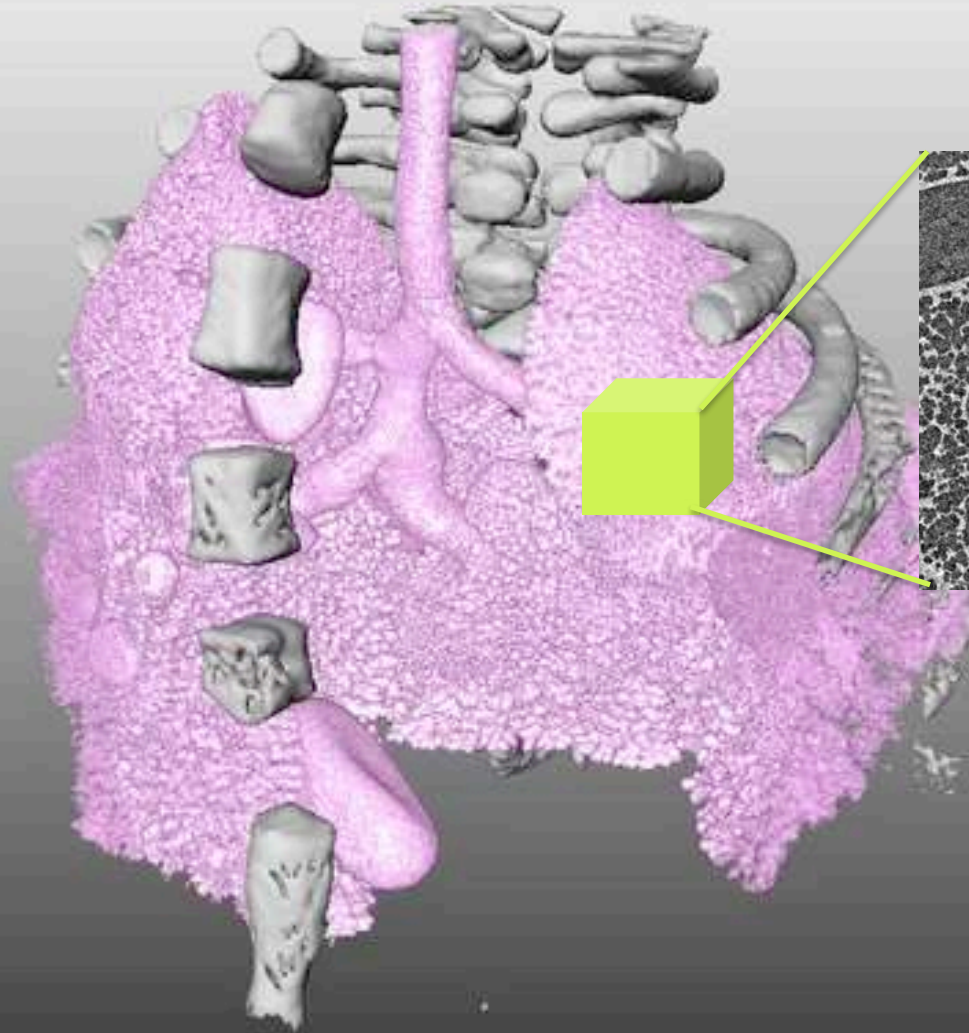
Space for the missing tools



Challenges

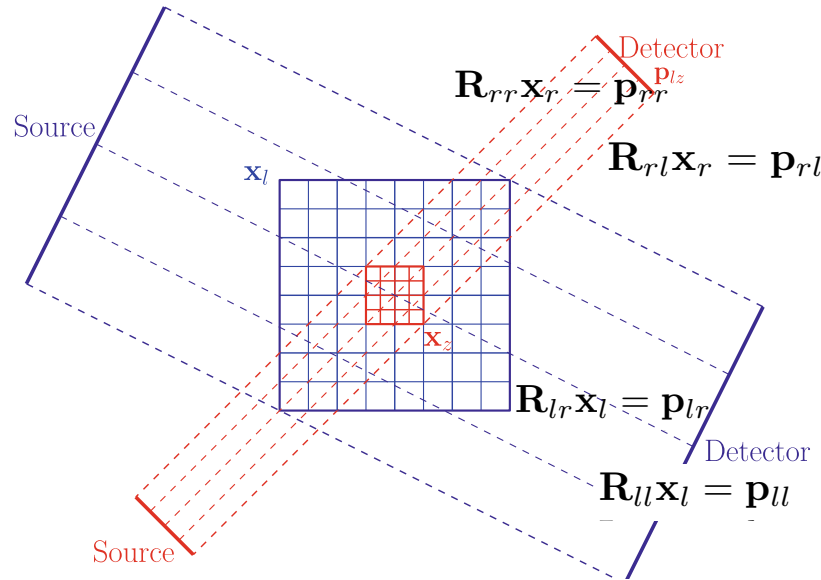
small field of view – large sample

High spatial resolution in large samples



Local tomography: a multiresolution approach

Iterative reconstruction from combined high-low resolution sinogram pairs



$$\mathbf{x}_r = \mathbf{y}_r + \mathbf{A}^T \mathbf{x}_l$$

Minimization of the energy function
Which is convex =>
Adam gradient descent optimisation

Fidelity term in low. res.

Fidelity term in high. res.

$$\mathcal{E}(\mathbf{x}_l, \mathbf{y}_r) = \|\mathbf{R}_{ll}\mathbf{x}_l + \mathbf{R}_{rl}\mathbf{y}_r - \mathbf{p}_{ll}\|_2^2 + \|\mathbf{R}_{lr}\mathbf{x}_l + \mathbf{R}_{rr}\mathbf{y}_r - \mathbf{p}_{lr}\|_2^2 +$$

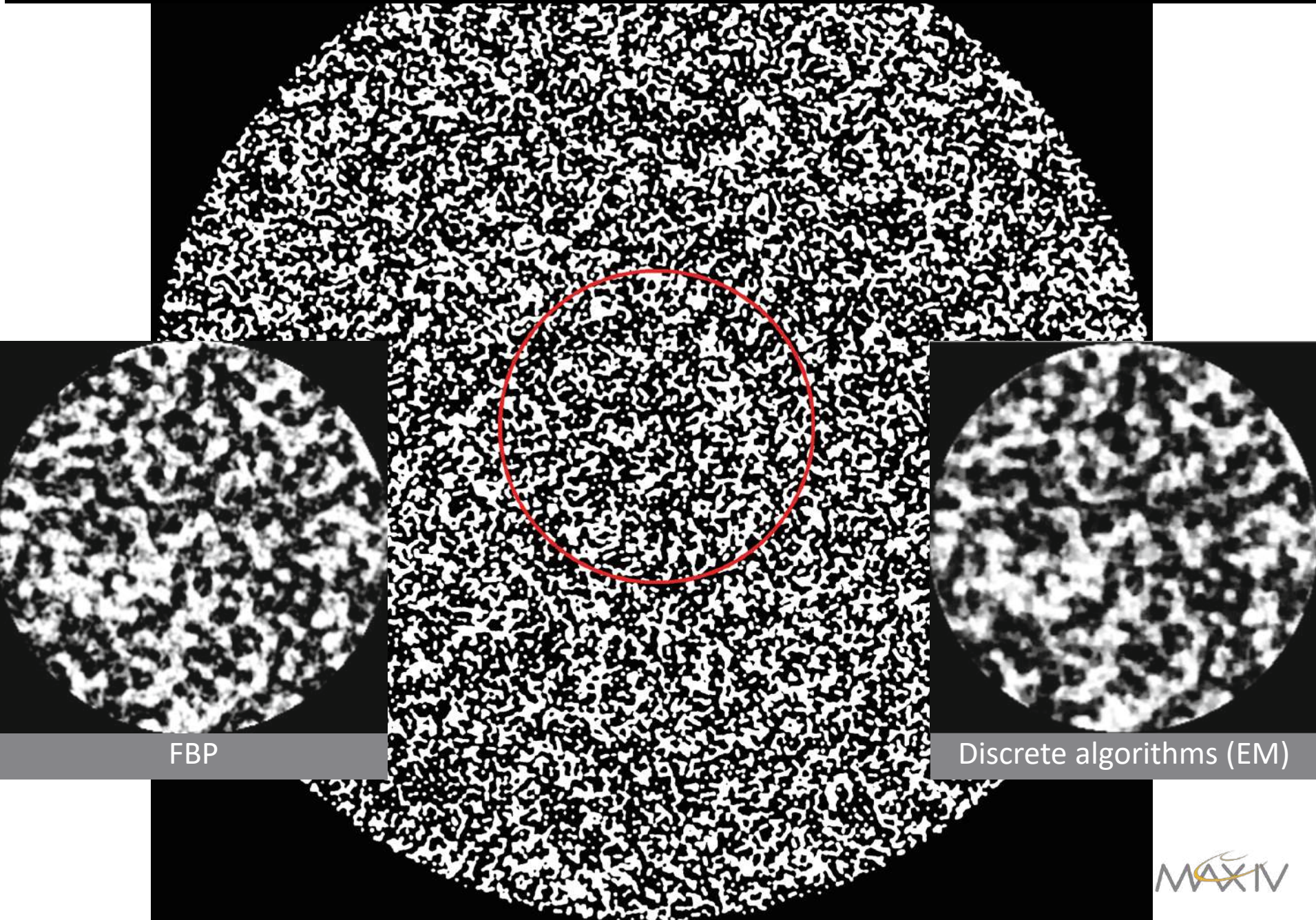
$$\gamma \|g(\mathbf{x}_l)\|_1 + \delta \|g(\mathbf{x}_r)\|_1 + \mu \|\mathbf{y}_r\|_1 + \nu \sum_{i=1}^{\omega_r^2} \sum_{j \in N_4(i)} |\mathbf{x}_r(i) - \mathbf{x}_r(j)|$$

positivity

Regularization of refinement image - noise

Smoothness regularization of 4-adjacent pixels of \mathbf{x}_r

Local tomography: a multiresolution approach



FBP

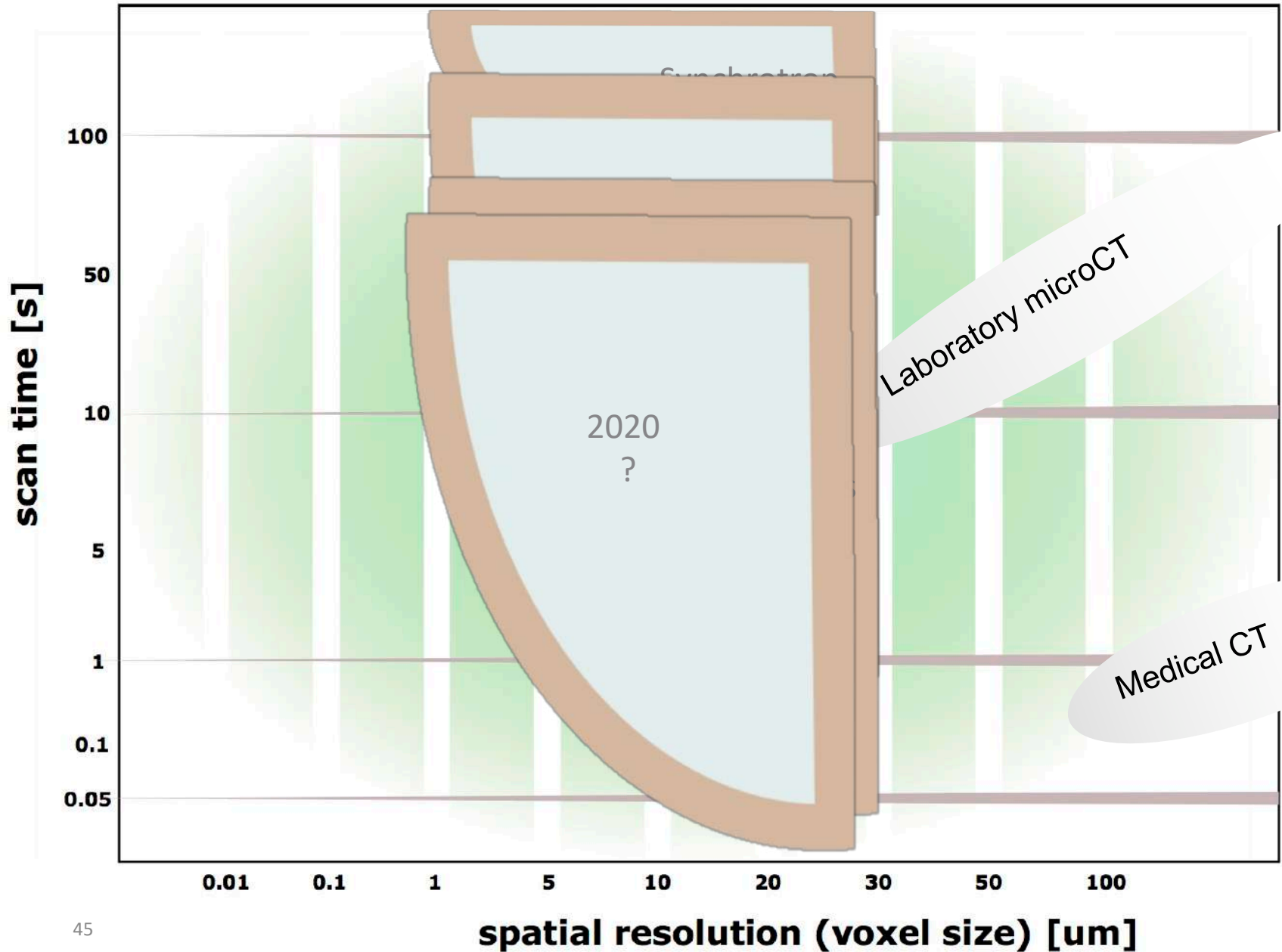
Discrete algorithms (EM)



Perspectives

New science?

Fast tomographic microscopy: 3D



Real life system dynamics

*The rise of the Aluminum foam:
From nucleation to film rupture
Acquisition speed: 208 tomo / s
TOMCAT beamline, SLS*

ARTICLE

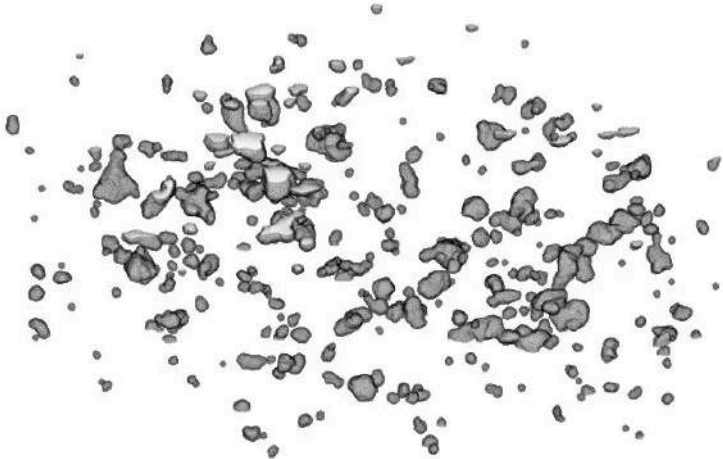
<https://doi.org/10.1038/s41467-019-11521-1>

OPEN

Using X-ray tomography to explore the dynamics of foaming metal

Francisco García-Moreno^{1,2}, Paul Hans Kamm^{1,2}, Tillmann Robert Neu^{1,2}, Felix Bülk^{1,2}, Rajmund Mokso³, Christian Matthias Schlepütz⁴, Marco Stampanoni^{4,5} & John Banhart^{1,2}

0.00 s

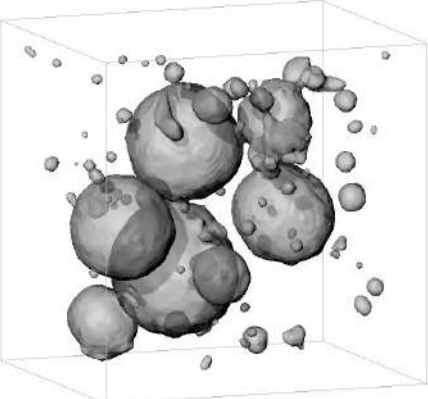


1 mm



Liquid Aluminum

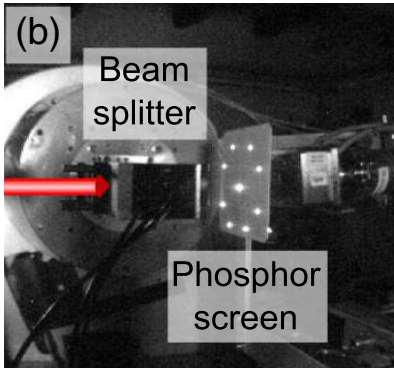
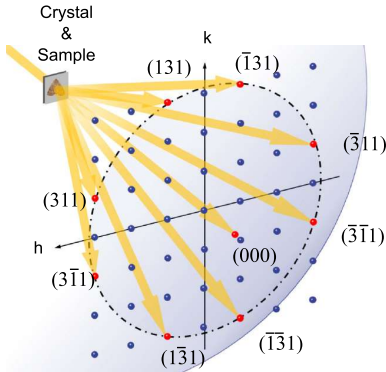
0 s



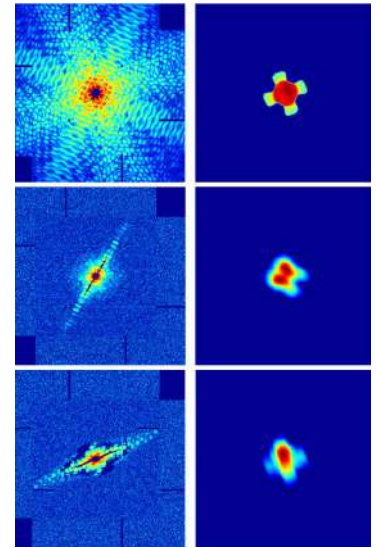
27.595000 s

Exploiting new geometries: multiprojection

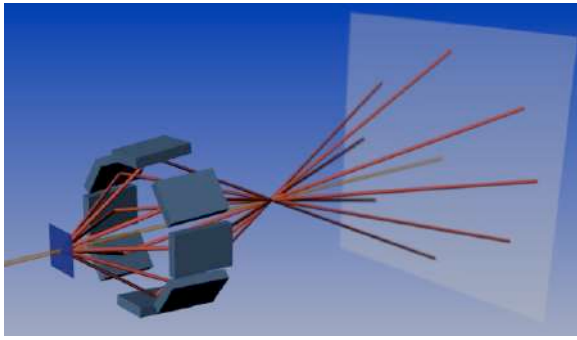
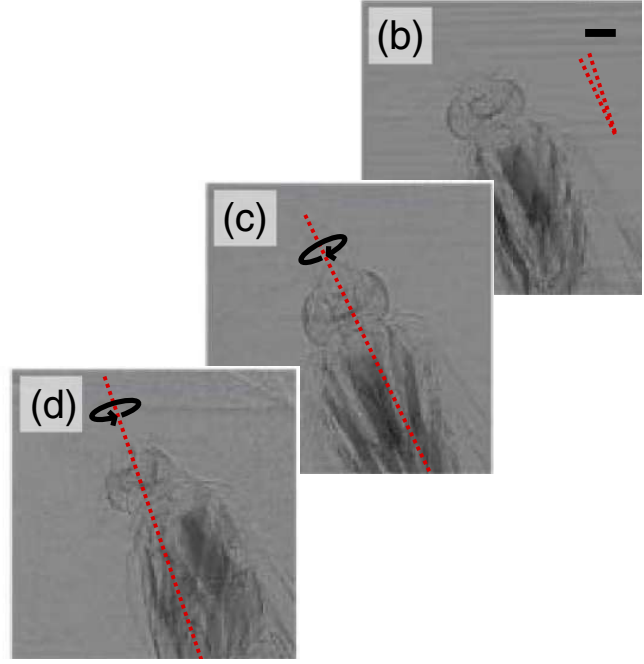
Realization of a 1:9 crystal splitter



Far-field diffraction



Near-field diffraction



Hard x-ray multi-projection imaging for single-shot approaches

P. VILLANUEVA-PEREZ,^{1,2,*} B. PEDRINI,¹ R. MOKSO,^{1,3} P. VAGOVIC,² V. A. GUZENKO,¹ S. J. LEAKE,⁴ P. R. WILLMOTT,¹ P. OBERTA,⁵ C. DAVID,¹ H. N. CHAPMAN,^{2,6,7} AND M. STAMPANONI^{1,8}

Joint project between Lund Uni, European XFEL, Berlin Uni.



Perspectives: shorter exposure times – less dose

Considerations for radiation dose optimization on the new sources:

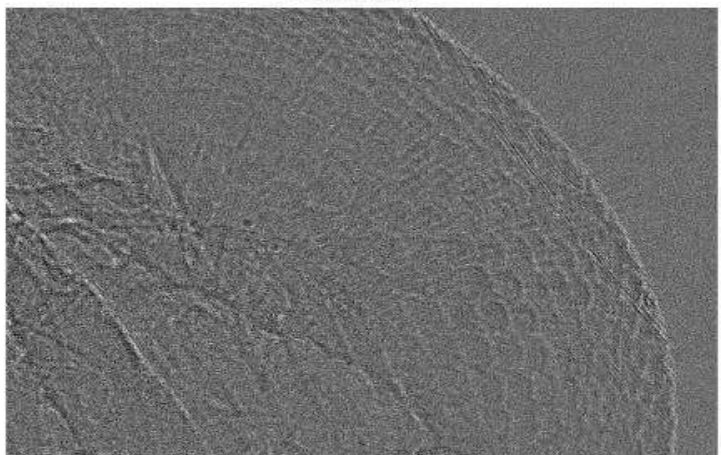
- ❖ *Fringe visibility is proportional to the transversal coherence length at the sample position (= source size and distance). The contrast (CNR or SNR) comes through detecting the oscillation(s) of the Fresnel pattern.*
- ❖ *Phase retrieval algorithms perform better for samples with weak (no) attenuation => it makes sense to increase X-ray energy until the attenuation can be neglected (<5%)*
- ❖ *Looking into most efficient photon detection schemes on the optics / detector side (e.g. simultaneous 2 distance acquisition)*
- ❖ *Make use of modern tomographic reconstruction algorithms*

Deep sub-micron in vivo imaging of
photoreceptor movement in drosophila
100 fps @ ID16B

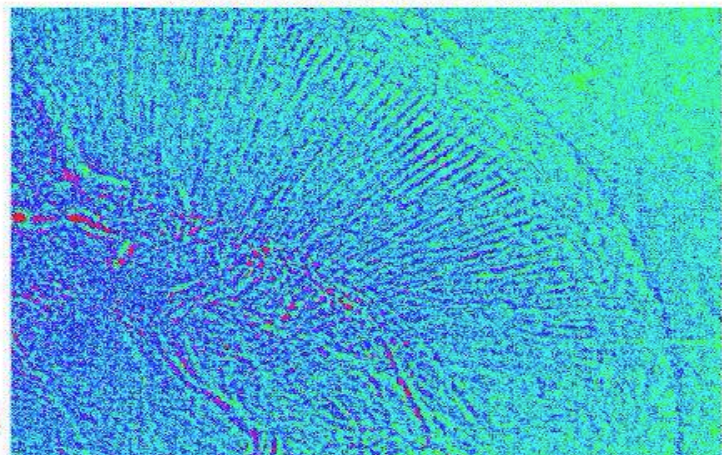
Mikko Juusola
University of Scheffield

Evidence for high resolution stereovision in compound eyes

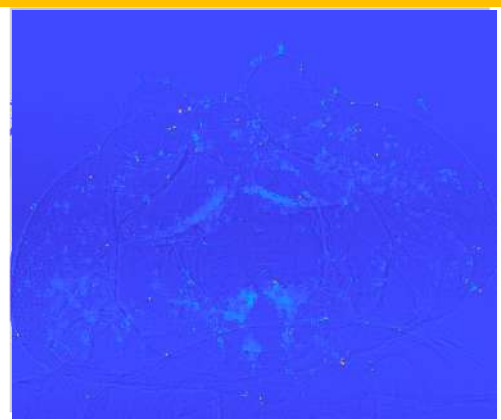
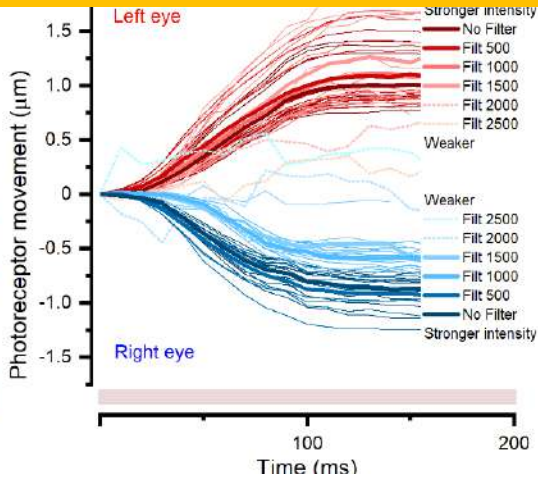
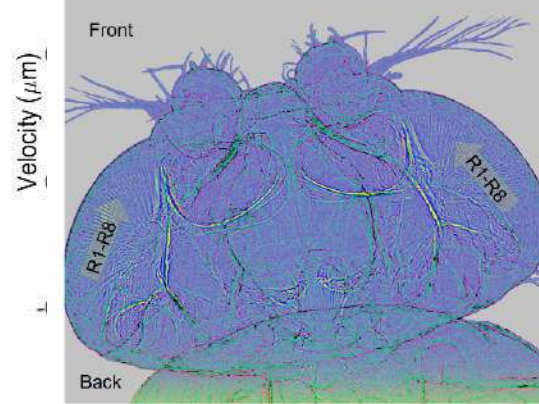
X-ray video

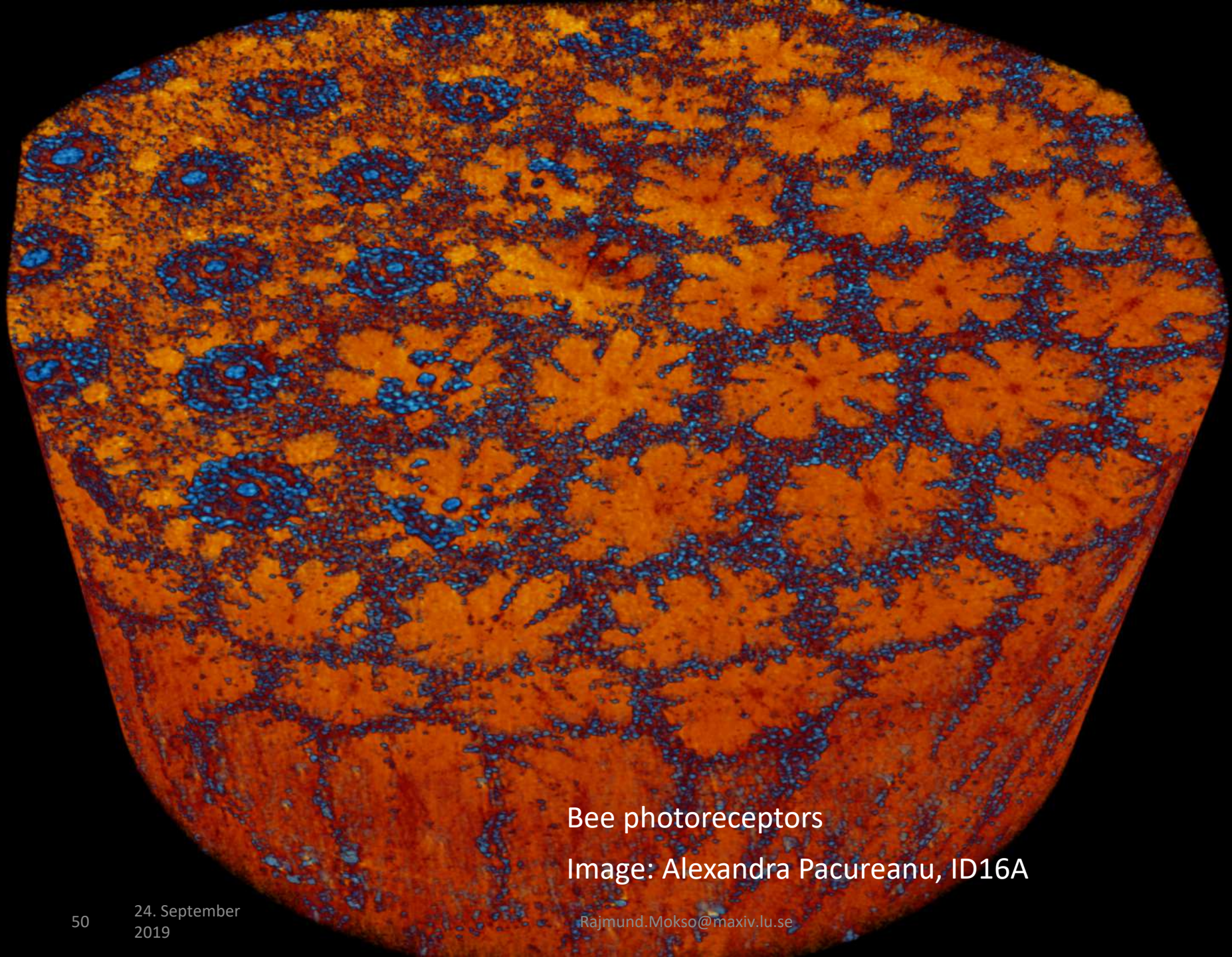


X-ray video with background removed



For reliable scientific interpretation In vivo measurement must be complemented by nanotomography with highest possible accuracy (cryo-nanotomo)





Bee photoreceptors

Image: Alexandra Pacureanu, ID16A

Perspectives: new science?

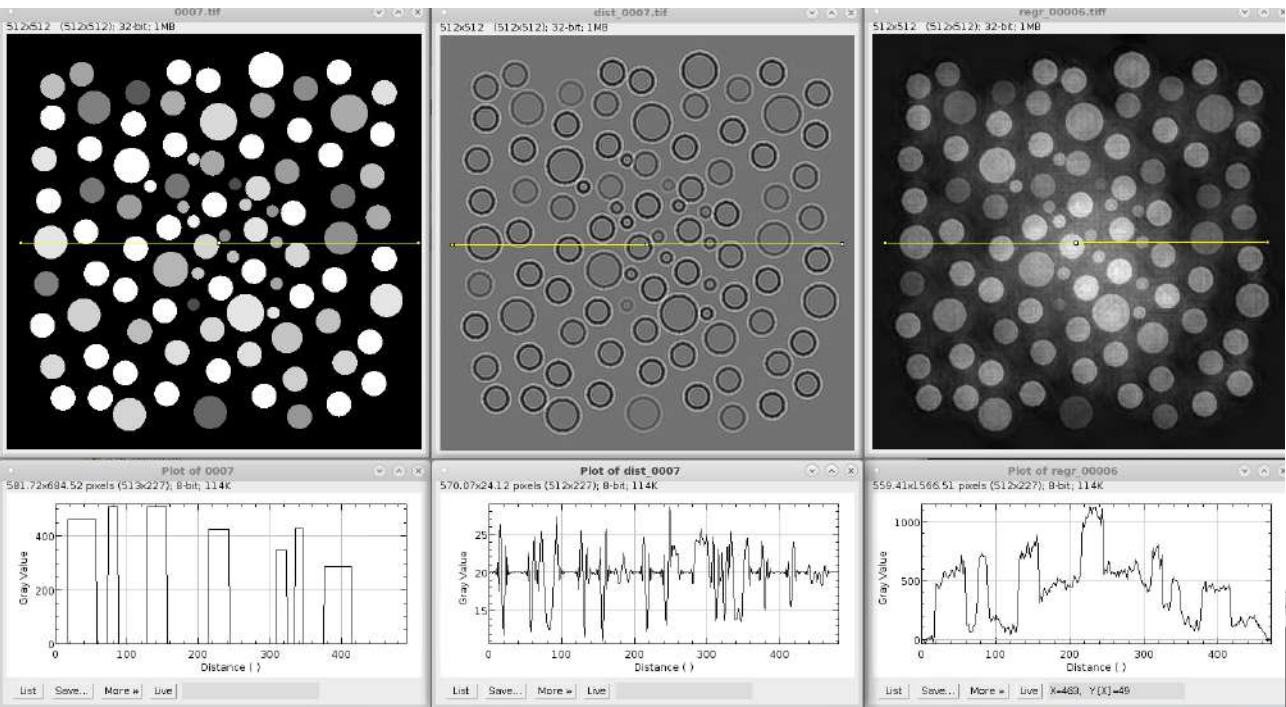
3D histology (in vivo)

- Image 3D living tissue at the cellular level in 3D

Nanoscale characterization of materials with realistic dimensions

- Pores, defects

Exploiting new phasing methods: convolutional neural networks



D. Figueras, summer project @ MAX IV

D. Pelt & J. Sethian, Mixed-scale dense CNN for image analysis, PNAS 115, 2018

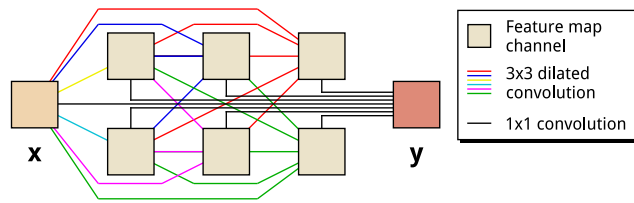
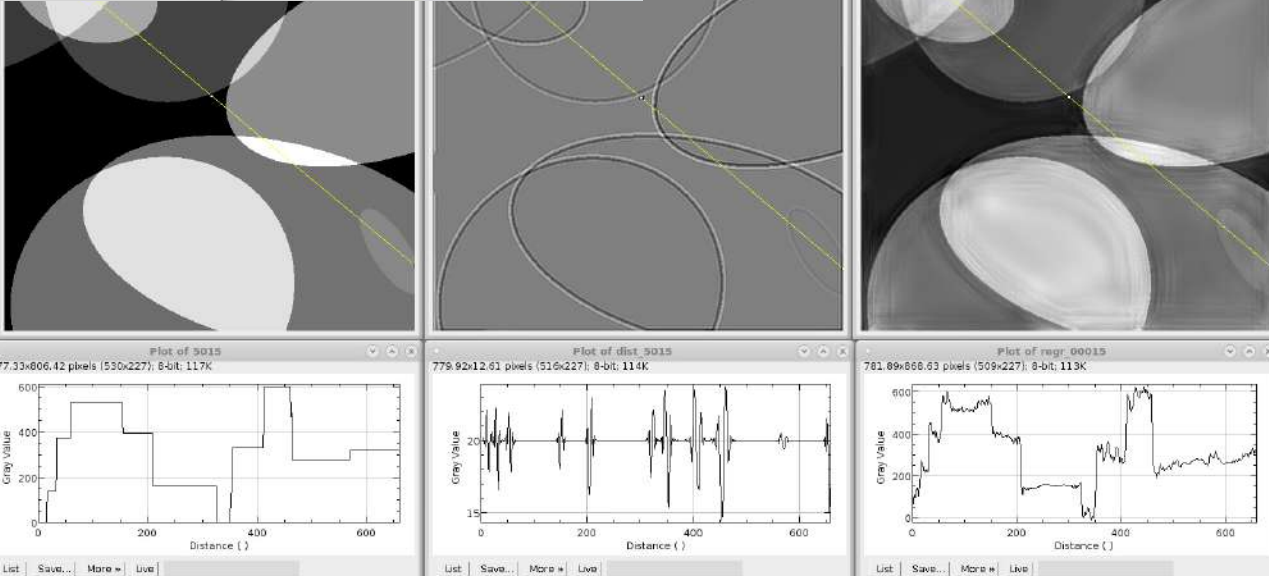


Fig. 3. Schematic representation of an MS-D network with $w=2$ and $d=3$. Colored lines represent 3×3 dilated convolutions, with each color representing a different dilation. Note that all feature maps are used for



Phase retrieval: Bayesian approach

Projection microscopy, ID16

Probability density functions are assigned to these quantities: $p(\mathbf{x})$ as the prior distribution, $p(\mathbf{y}|\mathbf{x})$ as the likelihood and $p(\mathbf{x}|\mathbf{y})$ as the posterior distribution. Applying the Bayes rule for probabilities [24], the posterior distribution is found to be proportional to the multiple of the previous two:

$$p(\mathbf{x}|\mathbf{y}) = \frac{p(\mathbf{y}|\mathbf{x}) p(\mathbf{x})}{p(\mathbf{y})} \propto p(\mathbf{y}|\mathbf{x}) p(\mathbf{x}), \quad (30)$$

where the denominator $p(\mathbf{y}) = \int p(\mathbf{y}|\mathbf{x}) p(\mathbf{x}) d\mathbf{x}$ is a normalizing constant called evidence, that will be ignored in the following formalism.

Let's consider a white noise distribution for ϵ , $p(\epsilon) = \mathcal{N}(\epsilon|0, v_\epsilon \mathbf{I})$, i.e. a Gaussian distribution with zero mean and where v_ϵ denotes the variance of the noise. In turn the likelihood follows a Gaussian distribution of mean equal to the model simulation $\mathbf{H}\mathbf{x}$ and variance equal to that of the noise:

$$p(\mathbf{y}|\mathbf{x}) = \mathcal{N}(\mathbf{y}|\mathbf{H}\mathbf{x}, v_\epsilon \mathbf{I}) \propto v_\epsilon^{-\frac{M}{2}} \exp\left\{-\frac{1}{2v_\epsilon} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2\right\}. \quad (31)$$

In the case of *maximum a posteriori* (MAP) estimation of the posterior distribution $p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y})$ the Bayesian approach [25] is able to infer on both the unknown quantity \mathbf{x} and the hyper-parameters of the model, $\boldsymbol{\theta}$:

$$\begin{aligned} \hat{\mathbf{x}} &= \arg \max_{\mathbf{x}} p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) = \arg \min_{\mathbf{x}} J(\mathbf{x}), & \text{where } J(\mathbf{x}) &= -\ln p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}), \\ \hat{\boldsymbol{\theta}} &= \arg \max_{\boldsymbol{\theta}} p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) = \arg \min_{\boldsymbol{\theta}} J(\mathbf{x}) \end{aligned} \quad (34)$$

Summary

With new sources we can explore a new spatio-temporal domain to understand structure and function of materials and bio samples

Full-field tomography with focused beam will profit the most from the new source. Some hope for further dose optimization also with parallel beams

The main bottleneck remains data reconstruction and analysis (phase retrieval, quantification)

To enable new science with new sources in place we need to invest more in bringing the data handling on the same level by (i) bringing existing tools into routine use and (ii) exploiting new avenues



Thank you!