Analytical considerations for reducing the emittance with longitudinally variable bends

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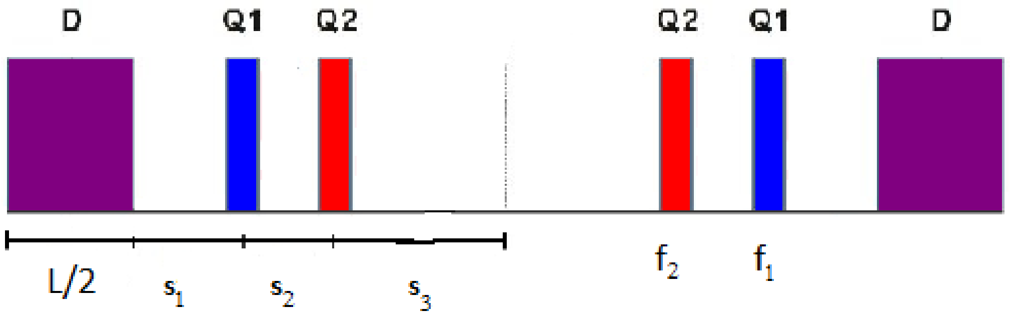
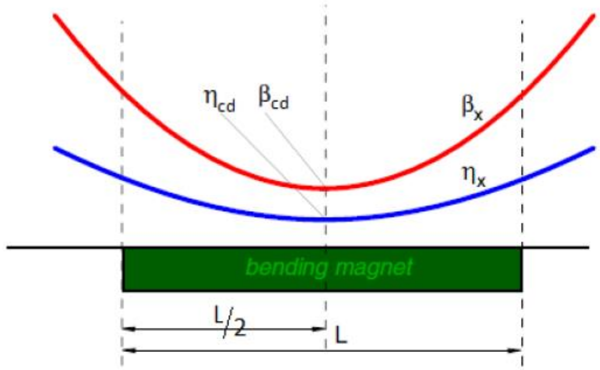
TME cell





$$L_{cell} = 2(s_1 + s_2 + s_3 + 2l_q) + L$$

The balance between radiation damping and quantum excitation results in the equilibrium betatron emittance. Using a theoretical minimum emittance, TME cell, low emittance values can be achieved. The horizontal emittance of the beam can be generally expressed as:

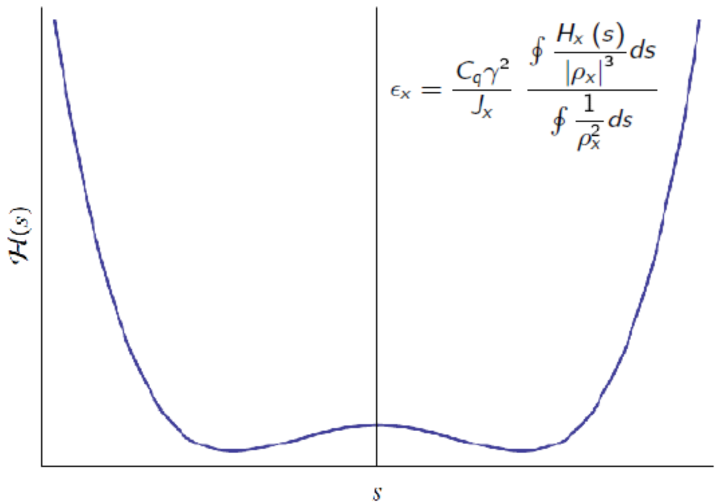
$$\epsilon_x = \frac{C_q \gamma^2}{J_x} \frac{\langle \frac{H_x}{|\rho_x|^3} \rangle}{\langle \frac{1}{\rho_x^2} \rangle} = \frac{C_q \gamma^2}{J_x} \frac{\frac{1}{C}\int_0^C \frac{\mathcal{H}_x}{|\rho_x|^3} ds}{\frac{1}{C}\int_0^C \frac{1}{\rho_x^2} ds}$$

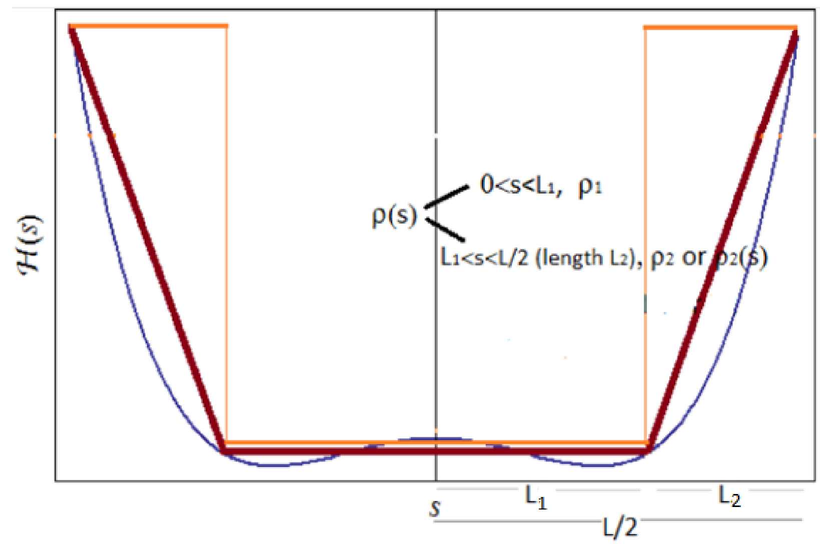
$$\beta(s) = \beta_{cd} - 2\alpha_{cd}s + \gamma_{cd}s^2,\ \alpha(s) = \alpha_{cd} - \gamma_{cd}s,\ \gamma(s) = \gamma_{cd},\ \eta(s) = \eta_{cd} + \eta'_{cd}s + \tilde{\theta}(s),\ \eta'(s) = \eta'_{cd} + \theta(s)$$

$$\mathcal{H}(s) = \gamma(s)\eta(s)^2 + 2\alpha(s)\eta(s)\eta(s)' + \beta(s)\eta(s)'^2$$

Longitudinally variable bends

Approaching the evolution of the uniform dipole's dispersion invariant assists in approaching its emittance behaviour in order to reduce it. The evolution of the dispersion invariant along the dipole guides the dipole profile choice for the emittance reduction.





Considering only the half dipole for simplicity (from 0 till L/2) as the other is symmetric and then dividing the dipole into two parts of different bending radii can be expressed as:

- length L_1 with bending radius $\rho_1(s),\ 0 < s < L_1$
- length L_2 with bending radius $\rho_2(s),\ L_1 < s < L_1 + L_2 = L/2$

<u>Bending angle of half dipole:</u>

$$\theta = \int_0^{L_1} \frac{1}{\rho_1(s)} ds + \int_{L_1}^{L_1+L_2} \frac{1}{\rho_2(s)} ds$$

References: J. Guo and T. Raubenheimer, (EPAC02), Y.Papaphilippou, P. Elleaume, PAC'05, R. Nagaoka, A.F. Wrulich, (NIM A575, 2007), C.-x Wang (PRST-AB, 2009)

Dipole profiles

Bending radii ratio $\rho = \frac{\rho_1}{\rho_2}$ Lengths ratio $\lambda = \frac{L_1}{L_2}$ $(\rho < 1 \ as \ \rho_2 > \rho_1 \ and \ \lambda > 0 \ as \ L_1, L_2 > 0)$

Emittance reduction factor

$$F_{TME} = \frac{\epsilon_{TME_{uni}}}{\epsilon_{TME_{var}}} \qquad F_{TME} > 1$$

FTME depends only on ρ and λ as the bending radii of the uniform and of the chosen profile are the same (as well as their length) and thus are simplified .

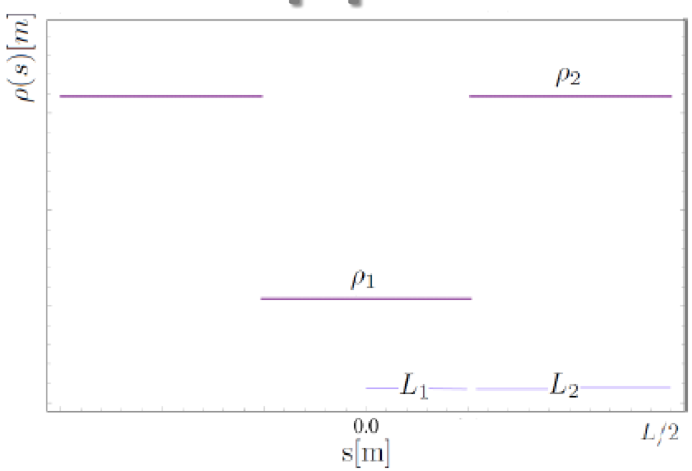
If the dipole's characteristics are not fixed FTME is a function of ρ and λ.

Fixing the dipole's characteristics in accordance to the design's **constraints** leads to the dependence of FTME either on ρ or λ . In this way the highest FTME value for a specific design can be found.

CLIC

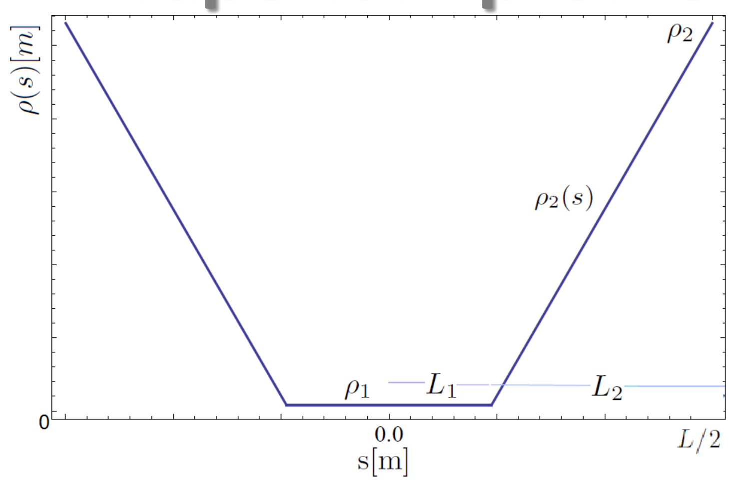
Bending angle of the dipole θ	Dipole's length L	Minimum bending radius ρ1 (maximum magnetic field 1.8T)
2π/100	0.6 m	5.4 m

Step profile



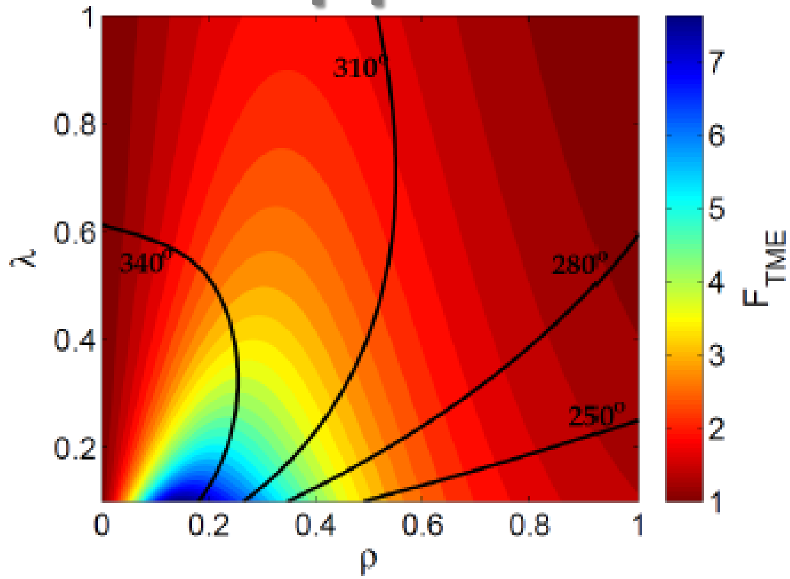
$$\rho(s) = \begin{cases} |\rho_1|, \ 0 < s < L_1 \\ \rho_2, \ L_1 < s < L_1 + L_2 \end{cases}$$

Trapezium profile

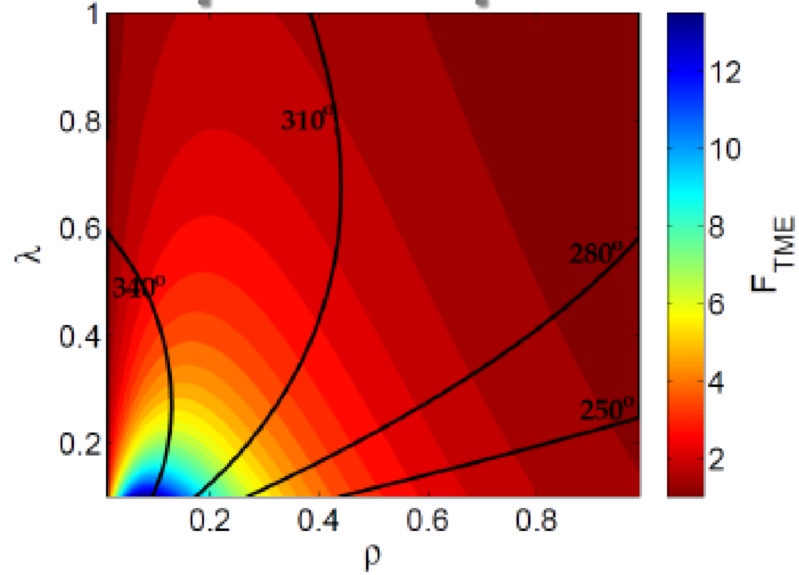


$$\rho(s) = \begin{cases} \rho_1, \ 0 < s < L_1 \\ \rho_2(s) = \rho_1 + (L_1 - s)(\rho_1 - \rho_2)/L_2, \ L_1 < s < L_1 + L_2 \end{cases}$$

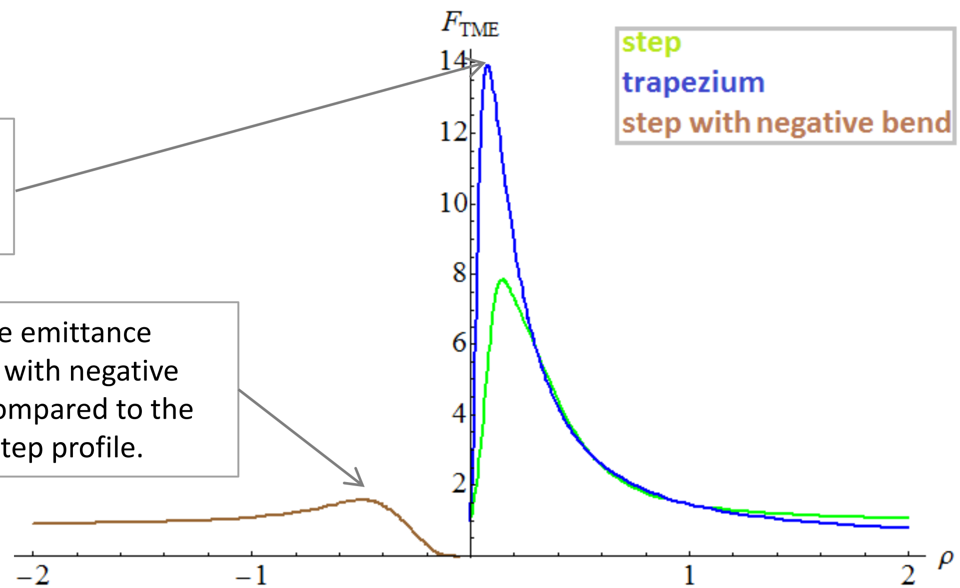
Step profile



Trapezium profile



The parameterization of the emittance reduction factor F_{TME} with the bending radii ratio ρ and the lengths ratio λ, always for λ>0.1 so that the lengths L_1, L_2 are comparable.
The black contour lines correspond to different values of horizontal phase advances (for the uniform dipole it is $\mu_{x,TME}$=284.5°).



The trapezium profile gives the highest emittance reduction

The maximum possible emittance reduction for the step with negative bend is insignificant compared to the one achieved for the step profile.

Comparison of the non-uniform dipole profiles' reduction factors when λ=0.1 (there the highest reductions are localized)

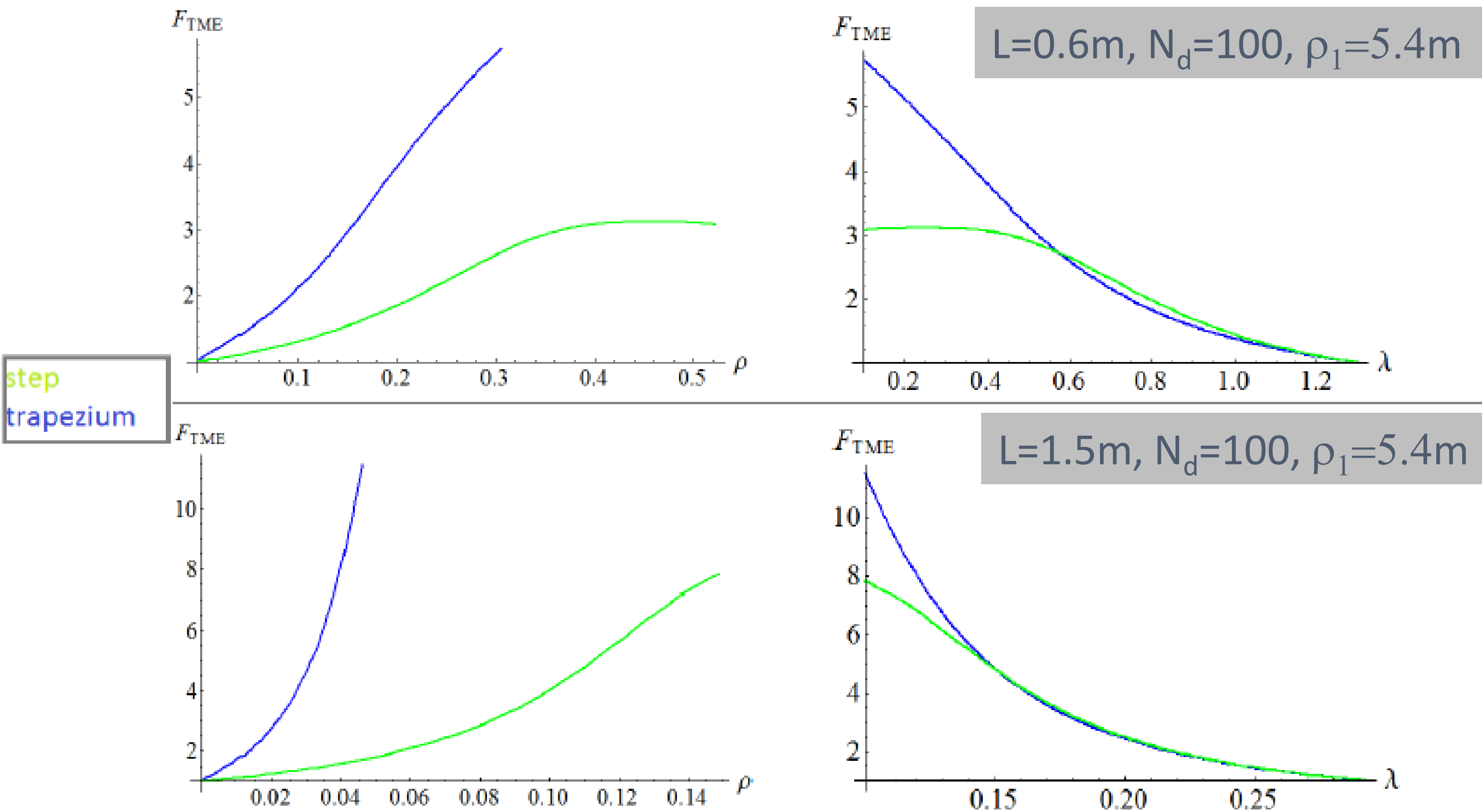
Comparison of non-uniform dipole profiles when fixing the dipole's characteristics (bending angle, length and minimum bending radius)

step → $\theta = \dfrac{L(\lambda+\rho)}{\rho_1(1+\lambda)}$

trapezium → $\theta = \dfrac{L(\lambda(-1+\rho)+\rho\log\rho)}{\rho_1(-1+\rho)(1+\lambda)}$

⇒ $\lambda(\rho)$ or $\rho(\lambda)$ ⇒ ($F_{TME}(\rho,\lambda)$) $F_{TME}(\rho)$ or $F_{TME}(\lambda)$

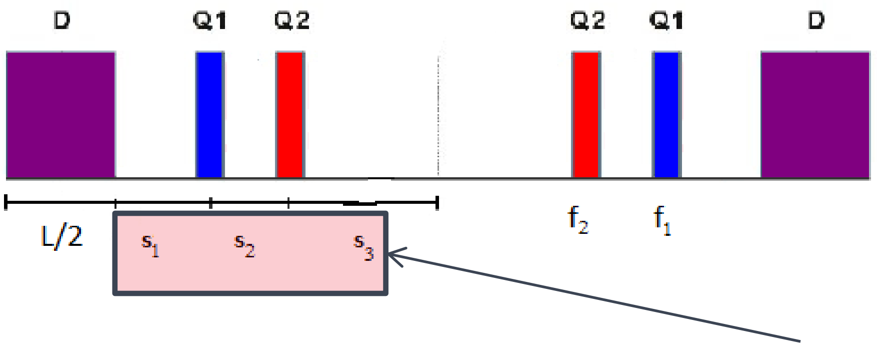
The reduction factor F_{TME} as a function of ρ (left) and as a function of λ (right)



Analytical parameterization of a variable bend TME cell (only for the trapezium profile)

Knowing the dipole's characteristics it is important to fix some more parameters in order to produce the numerical results for the CLIC DR lattice design:

- The quadrupoles' length is set to l_q = 0.2m.
- The maximum dipole field is set to 1.8T (minimum bending radius = 5.4m)
- The maximum pole tip field of the quadrupoles and the sextupoles is Bmaxq = 1.1T and Bmaxs = 0.8T respectively.
- The required output normalized emittance for N_d = 100 dipoles is 500nm and the operational energy of the CLIC Damping Rings complex of 2.86 GeV.



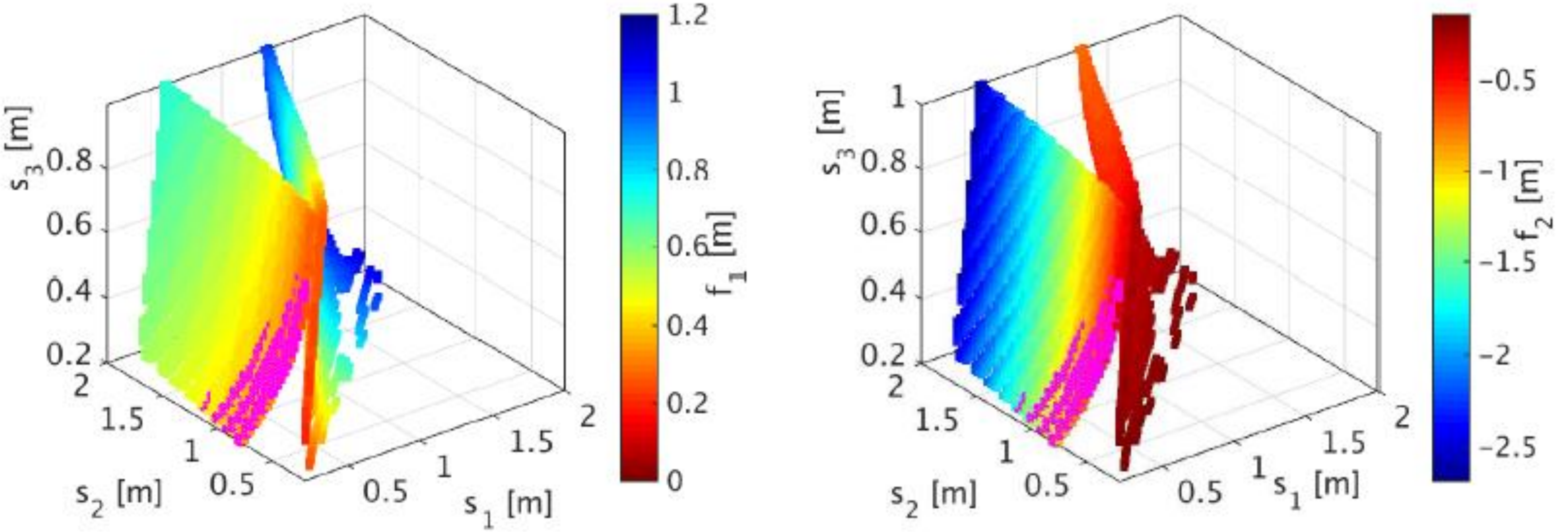
Fixing those parameters the free parameters left are the drift space lengths <u>s1, s2, s3</u> and the <u>emittance</u>. The stability criterion is governing every result and is included in the feasibility constraints:

$$|cos\varphi_{x,y}| < 1$$

$$k = \frac{1}{fl_q} =\leq \frac{1}{(B\rho_x)}\frac{B_q^{max}}{R_{min}}$$

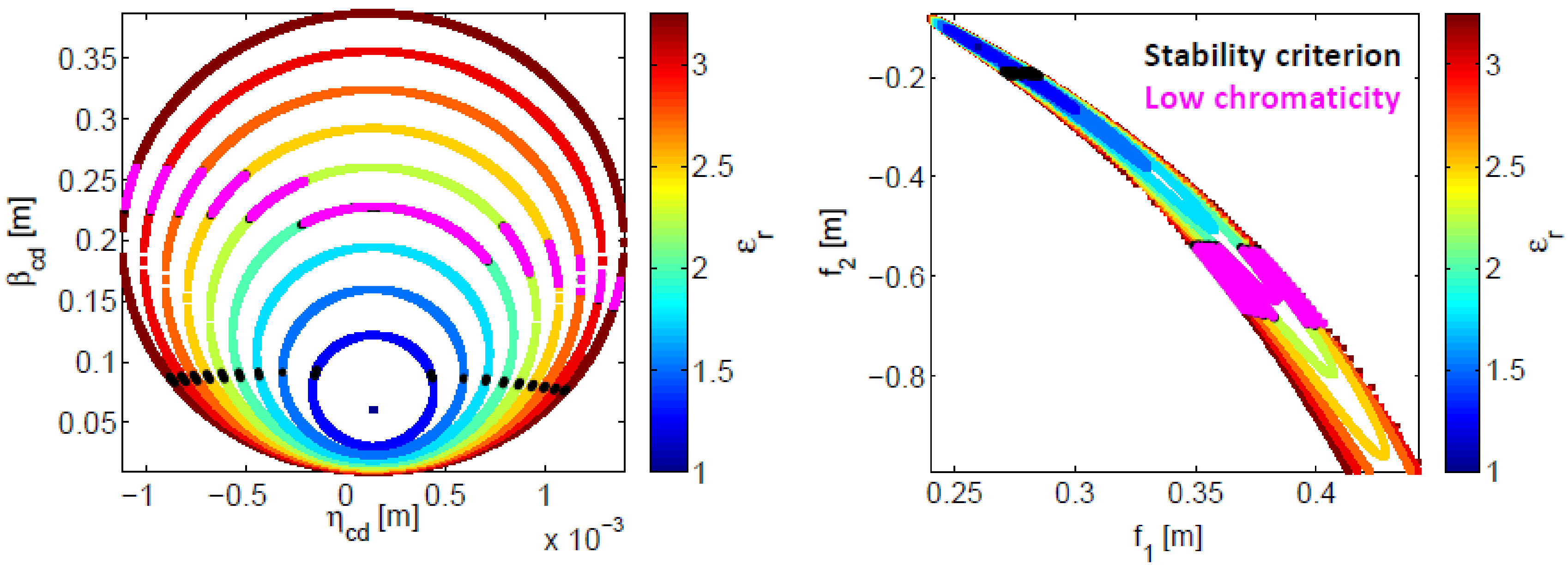
$$S \leq \frac{2B_s^{max}}{R_{min}^2}\frac{1}{(B\rho_x)}$$

Reference: F. Antoniou and Y. Papaphilippou, *PRSTAB*, **17**, 064002 , 23 June 2014



The focal lengths f_1 (Q_1 quadrupole) and f_2 (Q_2 quadrupole) are parameterized with the drift lengths s_1, s_2, s_3

Emittance detuning factor: Emittance deviation from the absolute TME $\epsilon_r = \frac{\epsilon_x}{\epsilon_{TME}}$



Parameterization of the beta and dispersion functions at the dipole center β_{cd}, η_{cd} and of the focal lengths f_1,f_2 with the detuning factor

Conclusions and next steps

Dipole profiles	Total cell's length Lcell [m]	FTMEmax (CLIC design)	λ(FTMEmax) (CLIC design)
Step	2.6	3.13	0.25
Trapezium	2.6	5.75	0.1

- The highest emittance reduction is given by the trapezium profile, concurrently it provides feasible-low chromaticity solutions for low detuning factors .
- The agreement with the simulation code MADX validates the analytical solutions for both profiles, specially for the thin lens approximation.
- Studies on the fringe fields created by the individual parts of the non-uniform dipoles will provide a better understanding of their behaviour.
- A further improvement of the final emittance values can be achieved when taking into consideration the collective effects, such as the Intrabeam scattering IBS that in the regime of ultralow emittances with high bunch charge has a significant impact on the emittance limits.

Paper in preparation: "Emittance reduction with variable bending magnet strengths: Analytical optics considerations"

Thank you!

Special thanks to F. Antoniou for her valuable help.

Dispersion invariant (1,2 for the individual dipole parts)

$$\mathcal{H}_{1,2}(s) = \gamma_{1,2}{\eta_{1,2}}^2 + 2\alpha_{1,2}\eta_{1,2}\eta'_{1,2} + \beta_{1,2}{\eta'_{1,2}}^2$$

$$(\alpha_{cd} = 0, \eta'_{cd} = 0)$$

Horizontal Emittance

$$\epsilon_x = G\left(\frac{1}{L_1}\int_0^{L_1}\frac{\mathcal{H}_1}{|\rho_1|^3}ds + \frac{1}{L_2}\int_{L_1}^{L_1+L_2}\frac{\mathcal{H}_2}{|\rho_2|^3}ds\right),\ where\ G = \frac{C_q\gamma^2}{J_x}\left(\frac{1}{L_1}\int_0^{L_1}\frac{1}{\rho_1^2}ds + \frac{1}{L_2}\int_{L_1}^{L_1+L_2}\frac{1}{\rho_2^2}ds\right)^{-1}$$

$$\epsilon_x = G\frac{(I_7 + I_8\lambda + (I_1 + I_2\lambda)\beta_{cd}^2 + \eta_{cd}(I_5 + I_6\lambda + (I_3 + I_4\lambda)\eta_{cd}))}{L_1\beta_{cd}}$$

$$I_1 = \int_0^{L_1}\frac{\theta_1^2}{|\rho_1|^3}ds\ 1,\quad I_2 = \int_{L_1}^{L_1+L_2}\frac{(\theta_2+\theta_{L_1})^2}{|\rho_2|^3}ds,\quad I_3 = \int_0^{L_1}\frac{1}{|\rho_1|^3}ds,\quad I_4 = \int_{L_1}^{L_1+L_2}\frac{1}{|\rho_2|^3}ds,\quad I_5 = \int_0^{L_1}2\frac{-s\theta_1+\tilde{\theta}_1}{|\rho_1|^3}ds$$

$$I_6 = \int_{L_1}^{L_1+L_2}2\frac{-s\theta_2+\tilde{\theta}_2 - L_1\theta_{L_1}+\tilde{\theta}_{L_1}}{|\rho_2|^3}ds,\quad I_7 = \int_0^{L_1}\frac{(-s\theta_1+\tilde{\theta}_1)^2}{|\rho_1|^3}ds,\quad I_8 = \int_{L_1}^{L_1+L_2}\frac{(-s\theta_2+\tilde{\theta}_2 - L_1\theta_{L_1}+\tilde{\theta}_{L_1})^2}{|\rho_2|^3}ds$$

Beta and dispersion functions for the TME

$$\beta_{TME} = \frac{\sqrt{-I_5^2 - 2I_5I_6\lambda + 4I_3(I_7+I_8\lambda) + \lambda(-I_6^2\lambda + 4I_4(I_7+I_8\lambda))}}{2\sqrt{(I_1+I_2\lambda)(I_3+I_4\lambda)}}\quad and\quad \eta_{TME} = -\frac{I_5+I_6\lambda}{2(I_3+I_4\lambda)}$$

$$\epsilon_{TME} = G\frac{(I_1+I_2\lambda)\sqrt{-I_5^2 - 2I_5I_6\lambda + 4I_3(I_7+I_8\lambda) + \lambda(-I_6^2\lambda + 4I_4(I_7+I_8\lambda))}}{L_1\sqrt{(I_1+I_2\lambda)(I_3+I_4\lambda)}}$$

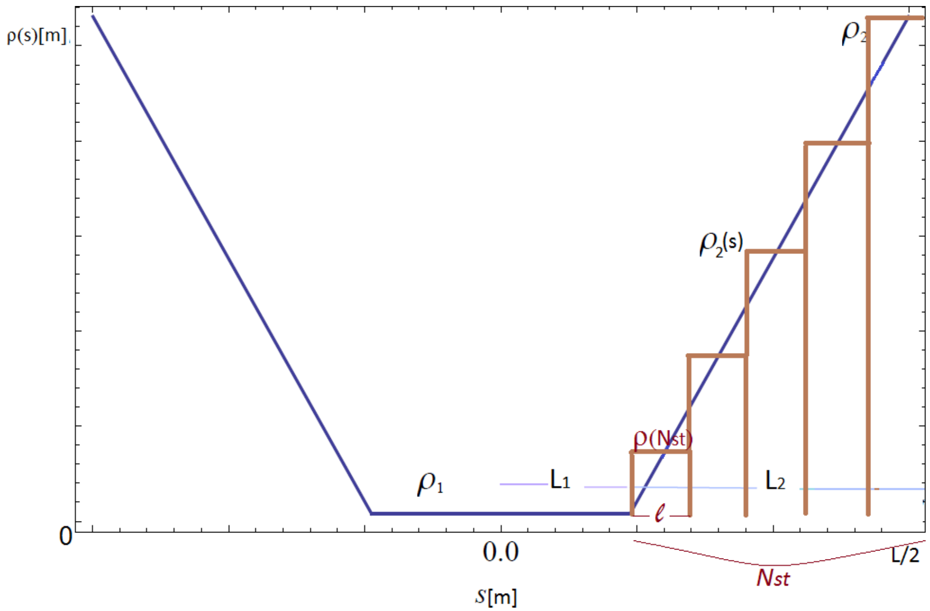
The relation between the reduction and the detuning factor.

$$\frac{\epsilon_{var}}{\epsilon_{uni}} = \frac{\epsilon_{r_{var}}\epsilon_{TME_{var}}}{\epsilon_{r_{uni}}\epsilon_{TME_{uni}}} = \frac{\epsilon_{r_{var}}}{\epsilon_{r_{uni}}}\frac{1}{F_{TME}}$$

$$\frac{\epsilon_{var}}{\epsilon_{uni}} < 1$$

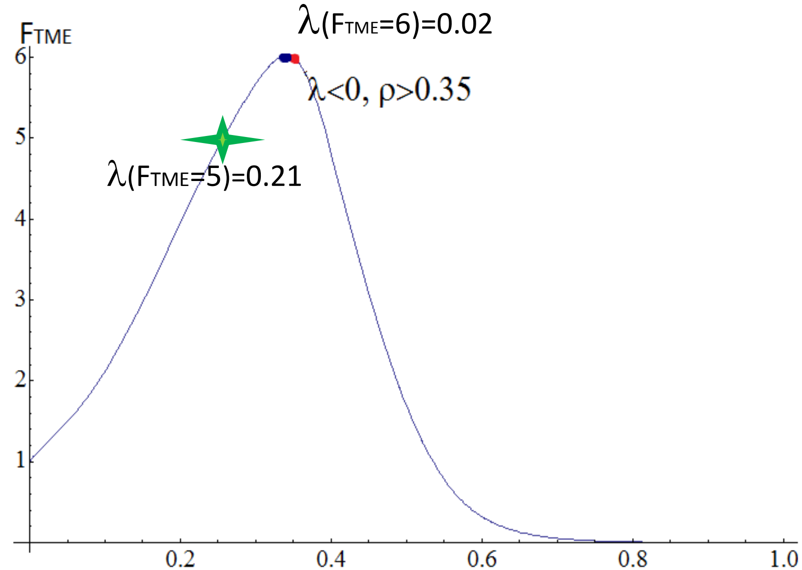
$$\Rightarrow\quad \frac{\epsilon_{r_{var}}}{\epsilon_{r_{uni}}} < F_{TME}$$

Comparison with MADX code for the trapezium profile



The dipole's bending radius evolution is being approached by a sequence of consecutive dipoles with the same length ℓ (their total number is Nst).

The number of steps used here to approach the bending radius evolution is Nst=10, increasing this number may seems to be a better approximation but actually the improvement is insignificant even when having Nst=100.



In order to get a good agreement with MADX a reduction $F_{TME}=5$ providing $\lambda=0.21$ is preferred.

The parameterization of the quadrupole strengths k_1, k_2 with the detuning factor is shown for quadrupoles length lq=0.01cm. The black colored solutions assure motion stability, the magenta colored areas give the obtainable MADX stable solutions for detuning factors $\varepsilon_r=2$ and $\varepsilon_r=3$.