Aberrations in curved x-ray multilayers

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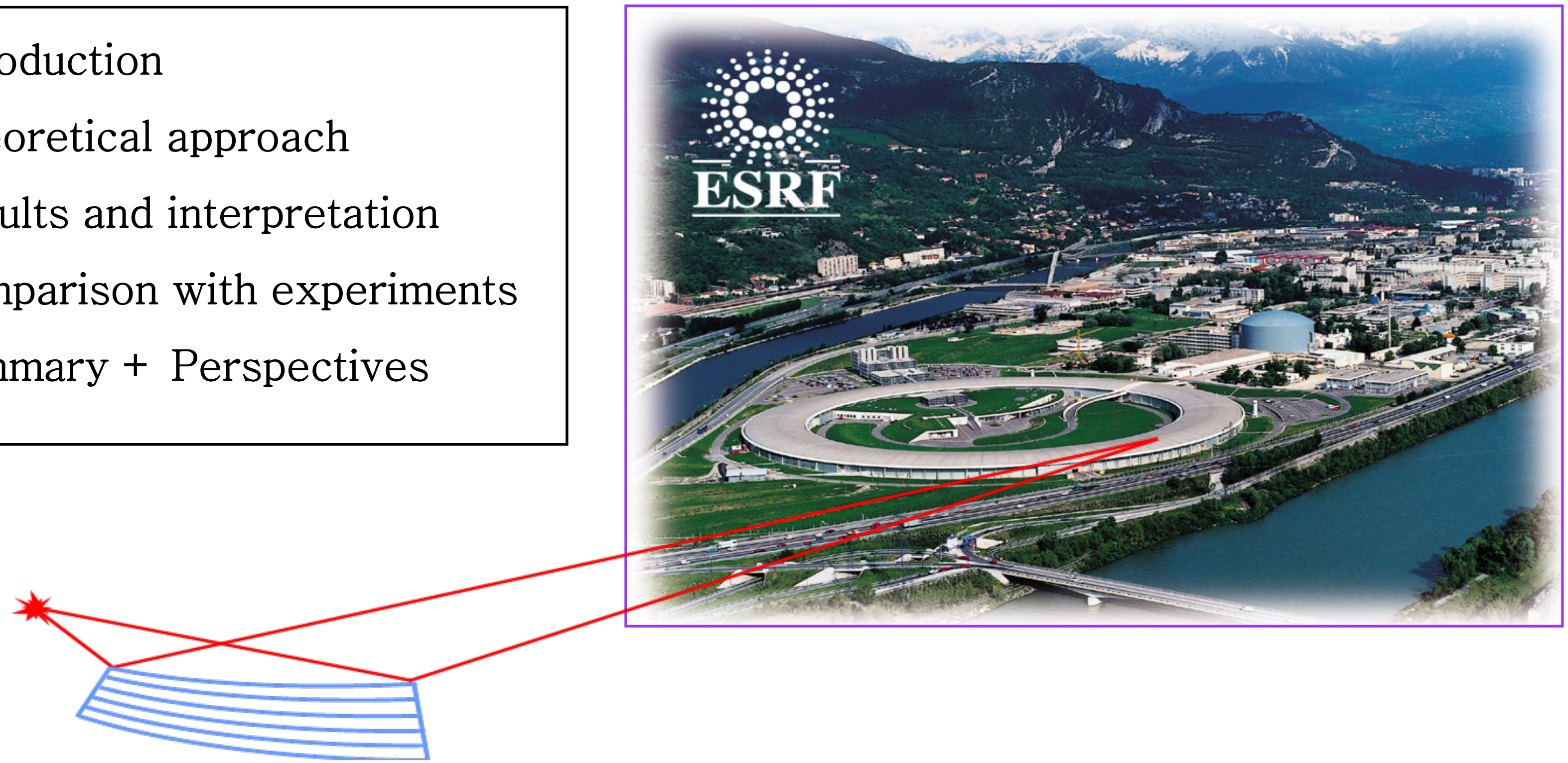
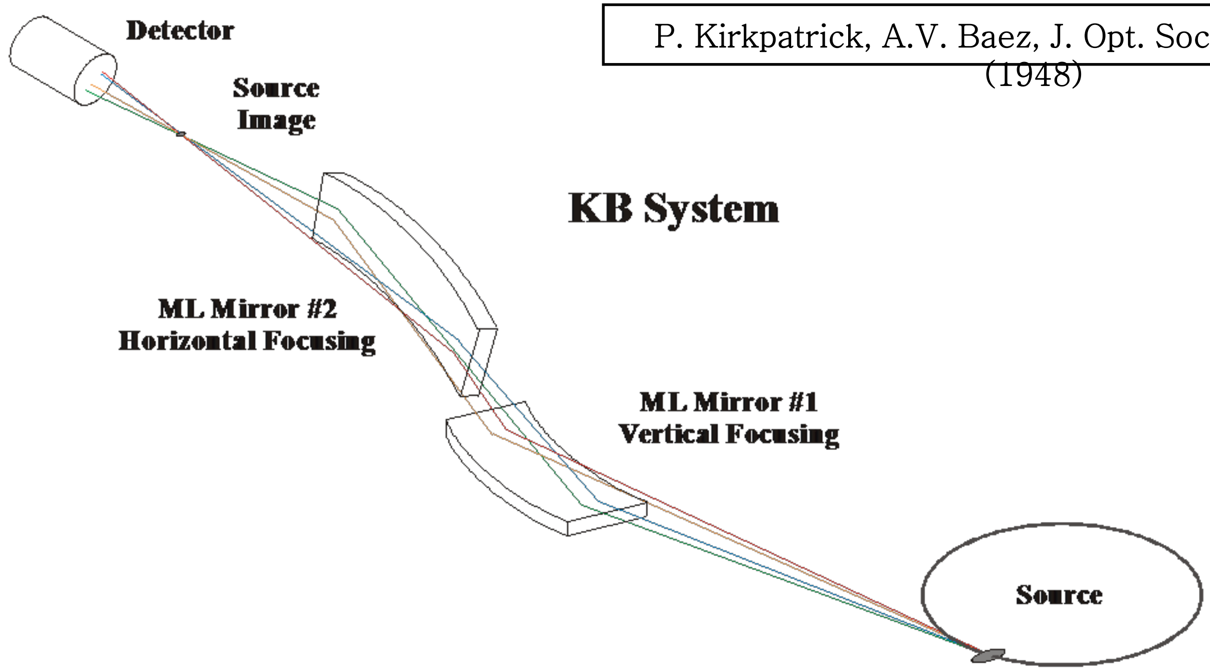


Introduction

Synchrotron optics: Kirkpatrick–Baez (KB) focusing devices

- Separate vertical and horizontal focusing (non-circular source)
- Technologically easier than single reflection ellipsoid + increased field of view
- Metal or graded ML coatings

P. Kirkpatrick, A.V. Baez, J. Opt. Soc. Am. 38, 766 (1948)



Introduction

Resolution: Simple approximation

Purely aperture limited

$$D_{FWHM} = \frac{0.44 \cdot \lambda}{NA}$$

Total reflection mirror

$$D_{FWHM} \approx 1.76 \cdot \sqrt{\frac{\pi}{r_0 \rho_e}}$$

$D_{FWHM} \approx$ 25 nm (Pt)

Multilayer mirror

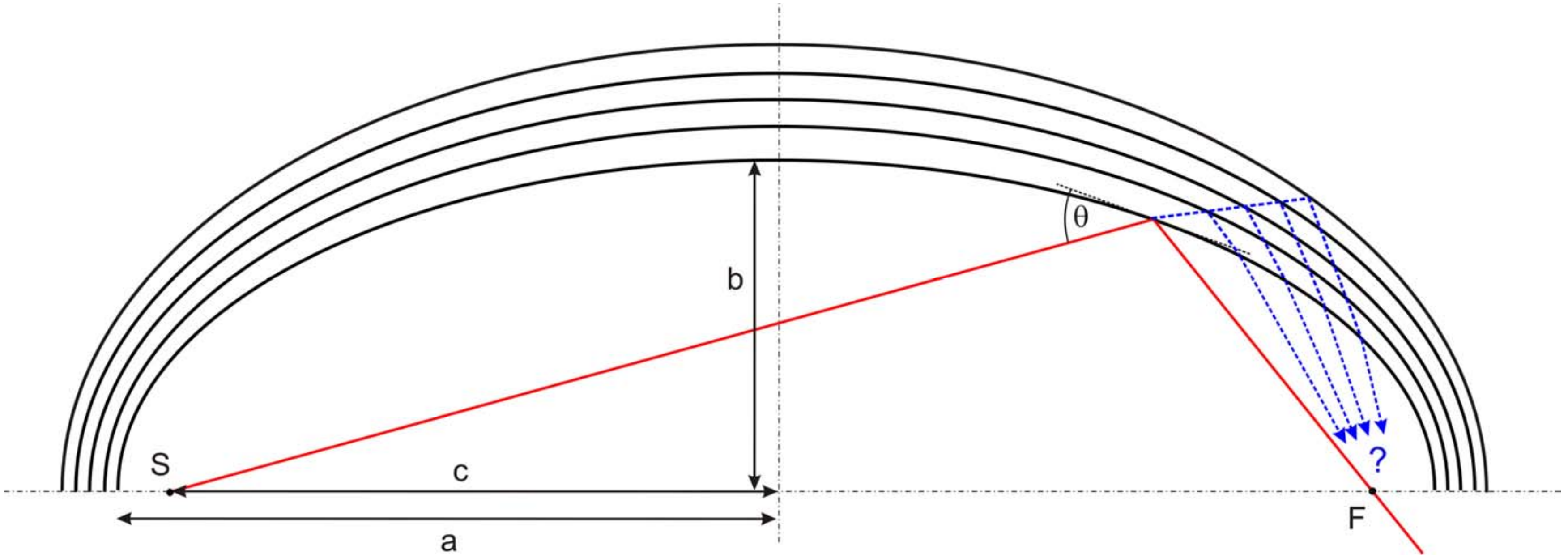
$$D_{FWHM} \approx \frac{0.88}{1/\Lambda_2 - 1/\Lambda_1}$$

$D_{FWHM} \approx$ 5 nm

No explicit energy dependence !

Theoretical approach

Basic geometry
- ML structure as nested ellipses (kinematical approximation)
- Upper surface reflection through ideal focus F
- Refraction for penetrating rays



Theoretical approach

Method

- ML ≅ Two-interface slab

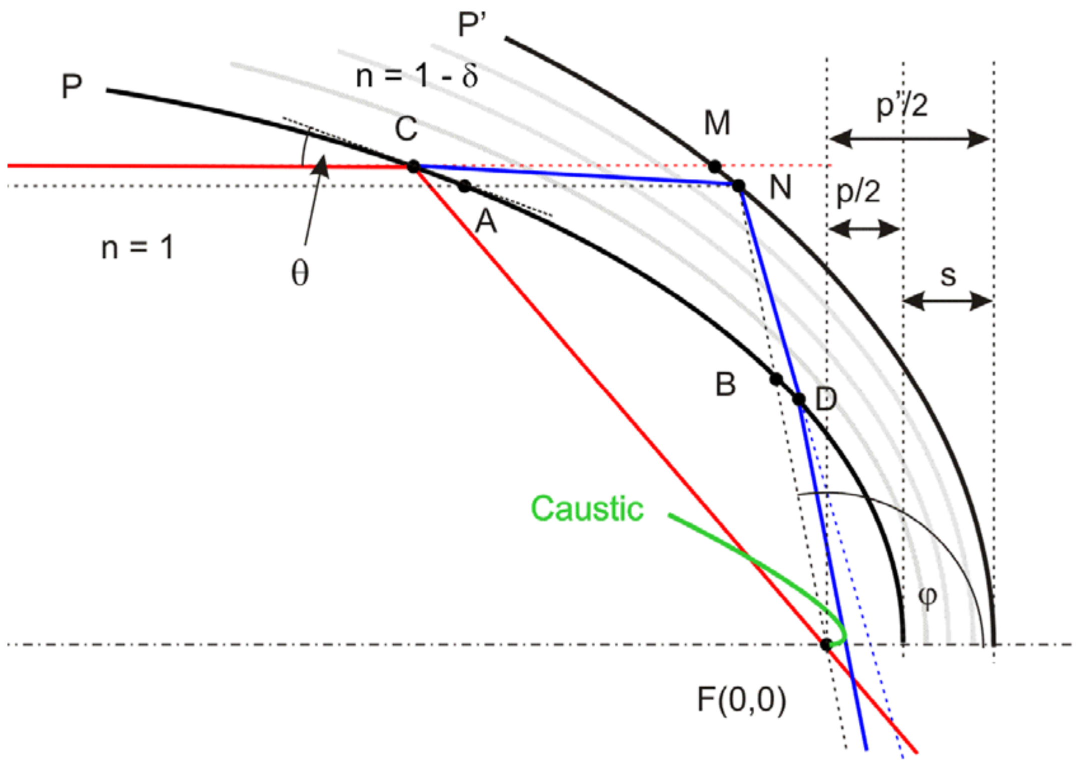
$$P\left(p/2,0\right), P'\left(p/2+s,0\right)$$

- Geometrical ray tracing
- Purely analytical approach
- Parametric representation

$$t=\tan\frac{\varphi}{2}\approx\frac{1}{\tan\theta}$$

Goals

- Caustic shape
- Beam intersections
- Chromatic behavior



J-P. Guigay et al, Opt. Express 16, 12050 (2008)

Theoretical approach

Approximations

- No multiple reflections
- Derivation to 1st order in δ

$$\Rightarrow \delta << 1$$

- Linear treatment of refraction

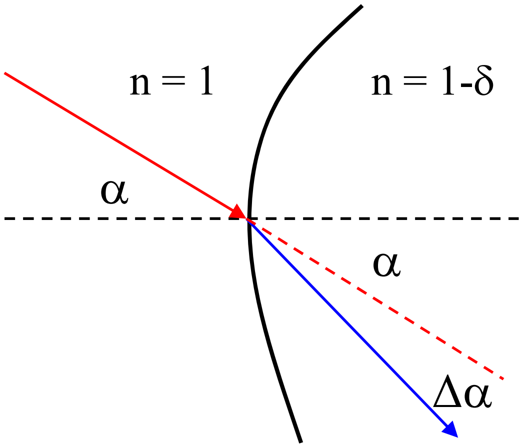
$$\Delta\alpha \approx \delta \cdot \tan\alpha$$

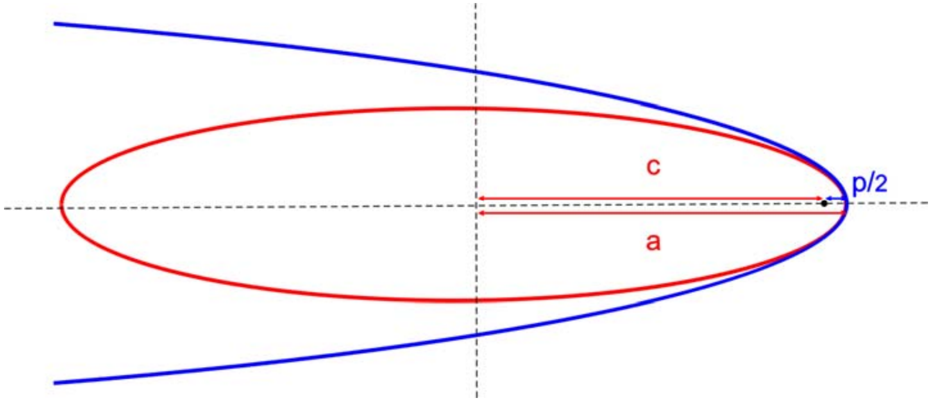
$$\Rightarrow \theta >> \theta_C$$

- Parabola ≅ eccentric ellipse

$$\frac{p}{2} \approx a - c$$

$$\Rightarrow e \approx 1$$





Theoretical approach

Penetration

- s given by mean extinction depth *z*

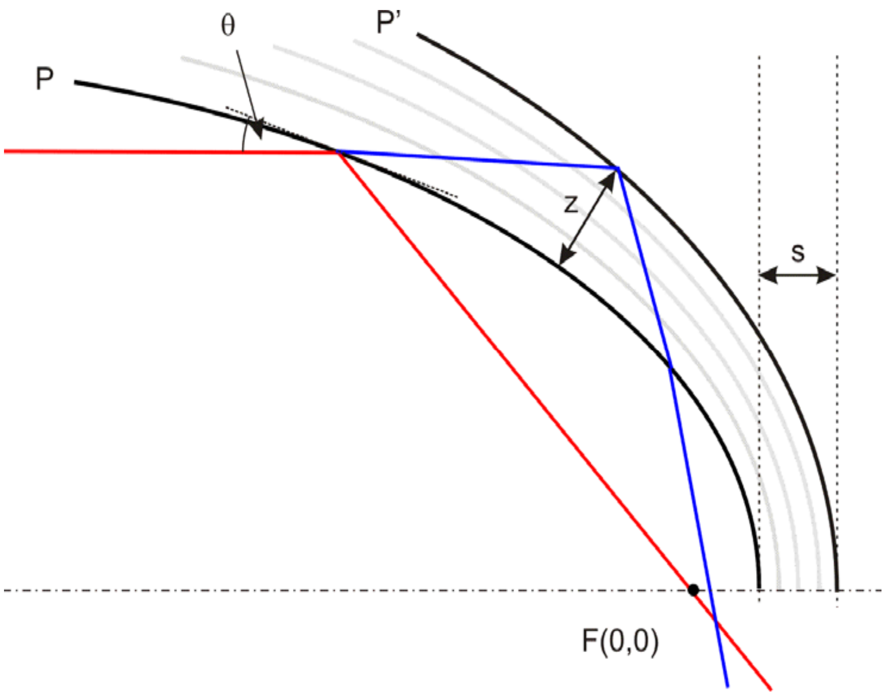
$$s \approx z(\theta)\cdot\sin\theta \approx const.$$

- Extinction depth estimated from flat MLs

$$z = N_C \cdot \Lambda$$

$$R(N_C) = \left(1 - \tfrac{1}{e}\right)\cdot R_{MAX}$$

Λ [nm]	θ	N_C	z [nm]	s [nm]
1.50	0.973°	163	244.5	4.113
2.00	0.735°	67	134.0	1.691
3.00	0.499°	24	72.00	0.6043
4.00	0.382°	13	52.00	0.3250
4.46	0.351°	10	44.60	0.2528
5.00	0.312°	8	40.00	0.1970
6.00	0.266°	5	30.00	0.1206
7.00	0.236°	4	28.00	0.0952



Data table: Flat [W/B_4C] ML at 24.55 keV

Results and interpretation

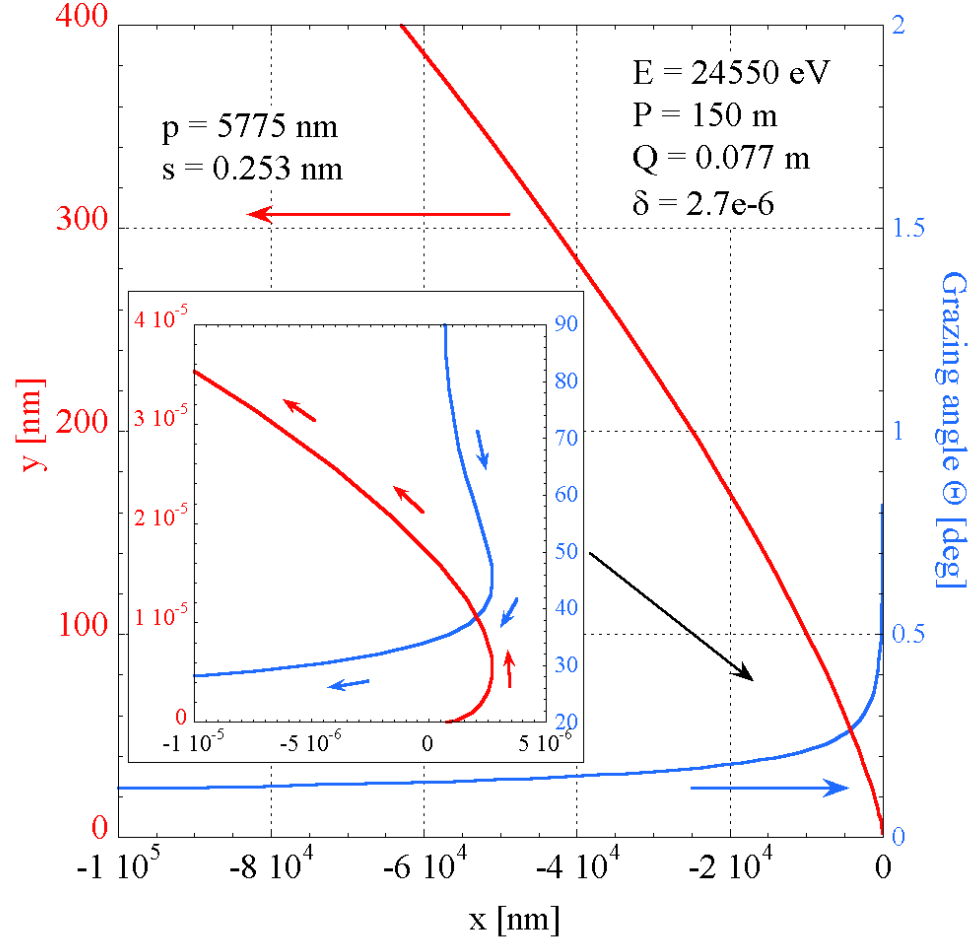
Caustic equations

$$x(t) = \delta \cdot s \cdot (1 + s/p) \cdot (1 + 6 \cdot t^2 - 3 \cdot t^4)$$

$$y(t) = \delta \cdot s \cdot (1 + s/p) \cdot 8 \cdot t^3$$

Results

- x and y diverge at grazing incidence
- δ and s amplify aberration effects
- No aberration for δ = 0 or s = 0
- Very small x-offset at normal incidence (Gaussian optics)



Results and interpretation

Beam intersections

$$X(Y=0) = \delta \cdot s \cdot \left(1 + 2 \cdot t^2 + t^4\right)$$

$$Y(X=0) = 2 \cdot \delta \cdot s \cdot \left(2 \cdot t + t^3\right)$$

Results

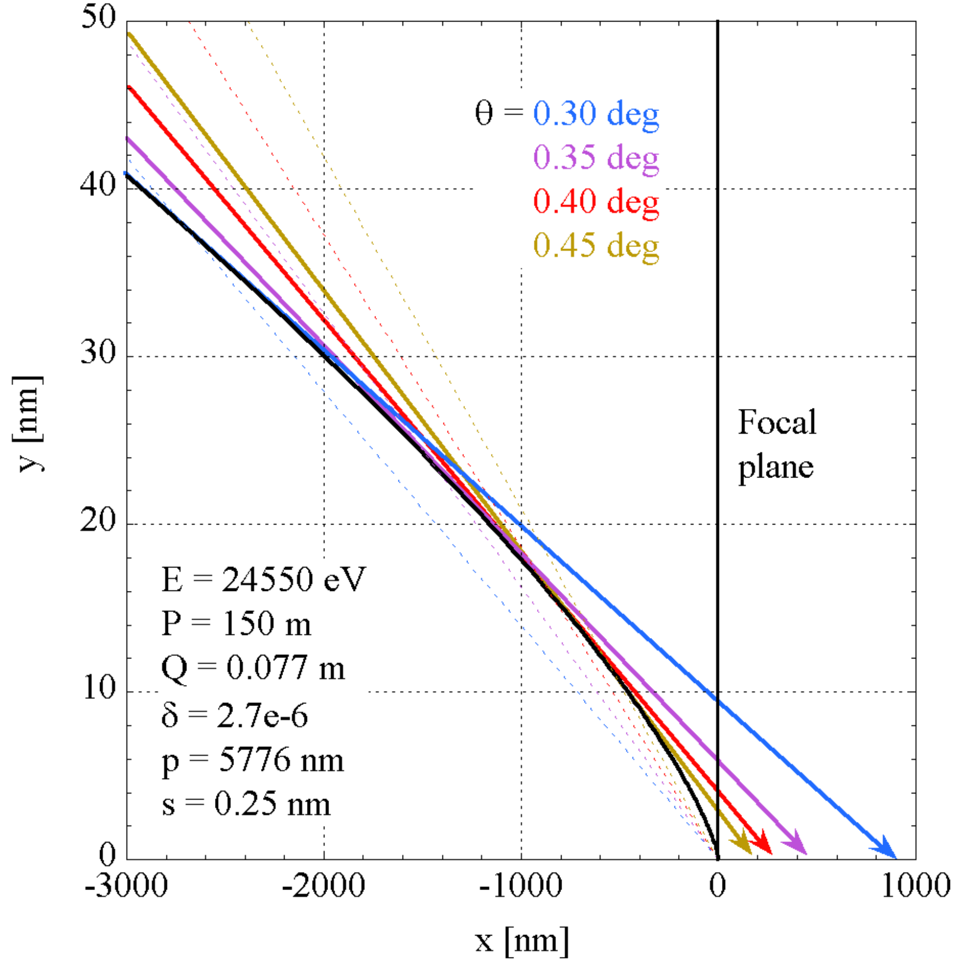
- Similar structure as caustic equations

→ similar conclusions

- Order of magnitude

  Δx ≤ 1000 nm

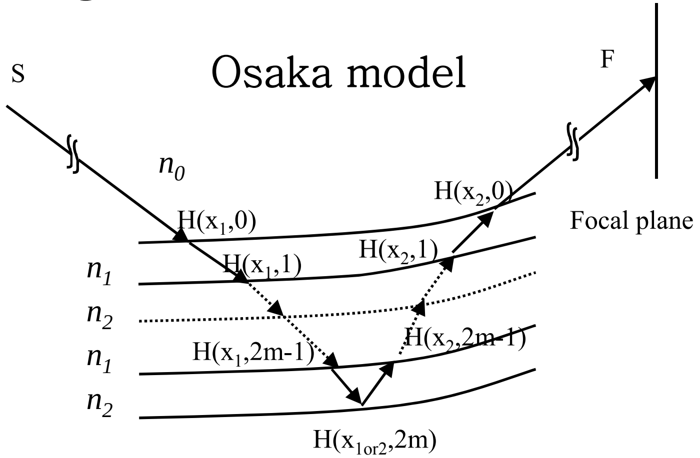
  Δy ≤ 10 nm

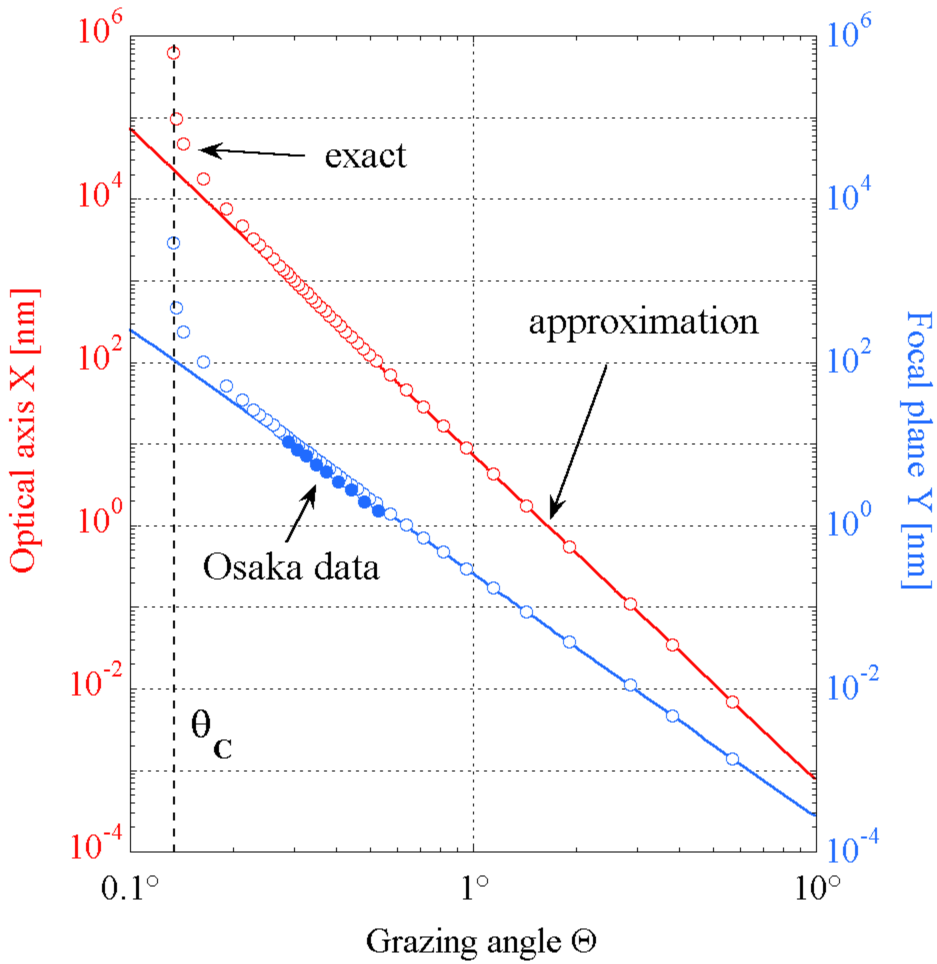


Results and interpretation

Comparison with exact ray tracing

- Snell's law
- No approximations
- Good agreement with analytical model
- Linear approach for refraction fails near critical angle
- Agreement with Osaka data





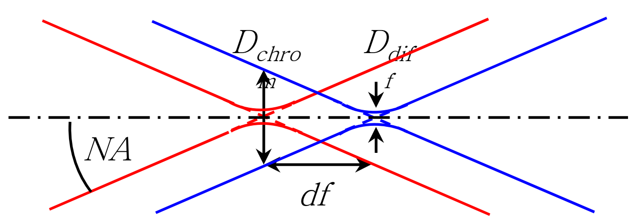
Results and interpretation

Chromaticity

$$\delta \propto \frac{1}{E^2} \longrightarrow \left|\frac{df}{dE}\right| = 2 \cdot \frac{\Delta f}{E}$$

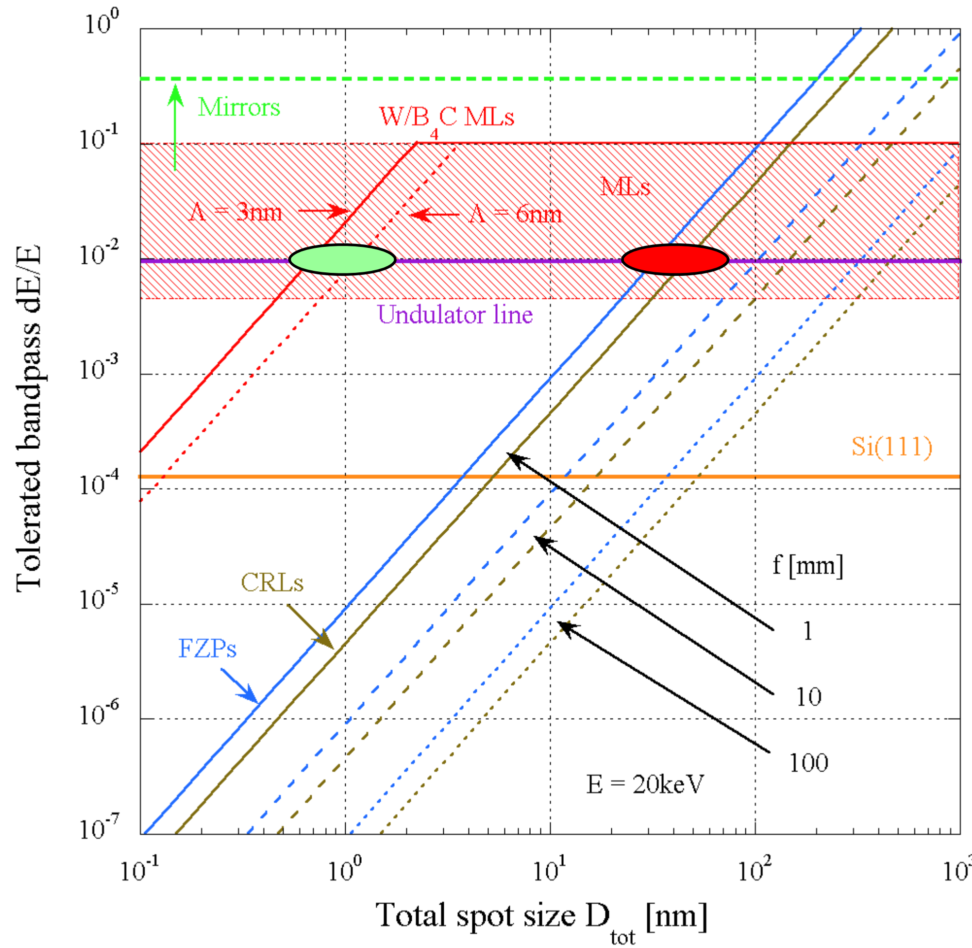
$$D_{tot}^2 = D_{diff}^2 + D_{chrom}^2 \overset{!}{\leq} 2 \cdot D_{diff}^2$$

$$D_{tot}^2 = 1.76 \cdot \lambda \cdot df$$



	FZP	CRL	RML
$\frac{dE}{E}$	$\frac{D_{tot}^2}{1.76 \cdot f \cdot \lambda}$	$\frac{D_{tot}^2}{3.52 \cdot f \cdot \lambda}$	$\frac{D_{tot}^2}{3.52 \cdot \Delta f \cdot \lambda}$

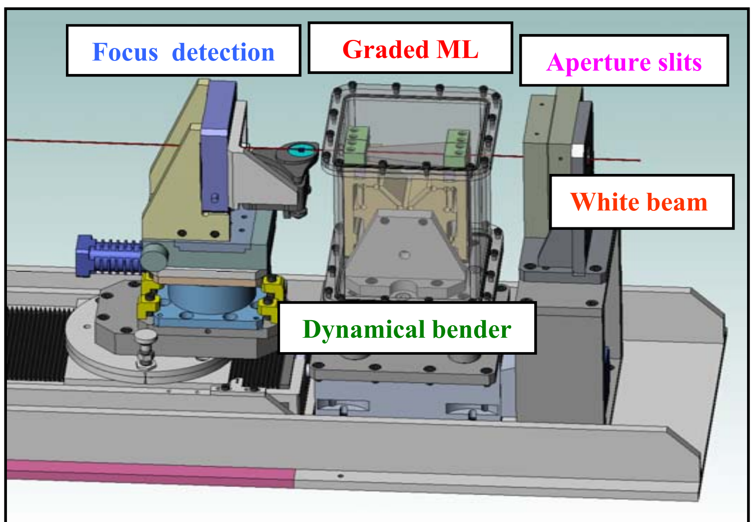
Secondary effect on focus !

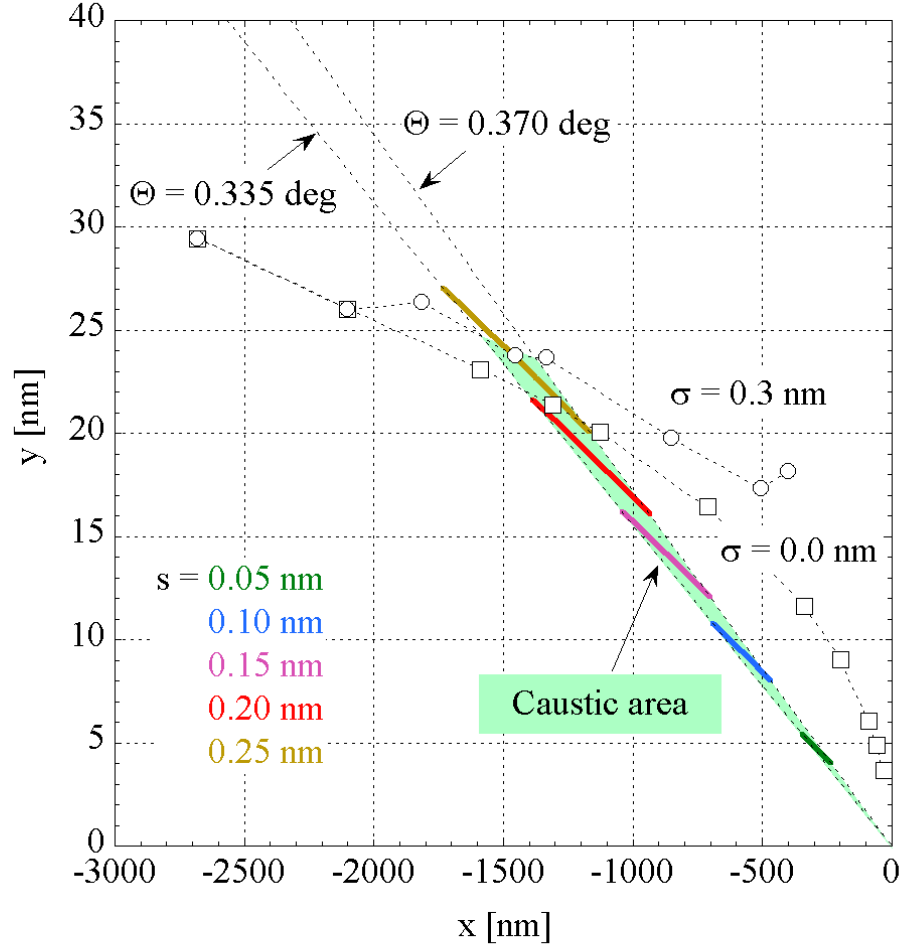


Comparison with experiments

Focusing experiment on ID19

- $[W/B_4C]_{25}$ ML @ 24550 eV
- P = 150 m , Q = 76.9 mm , e = 1 − $4 \cdot 10^{-8}$
- Angular aperture: 0.335º…0.370º
- Penetration: z ≈ 45 nm ≈ 10 periods
- p = 5776 nm , s = 0.2528 nm

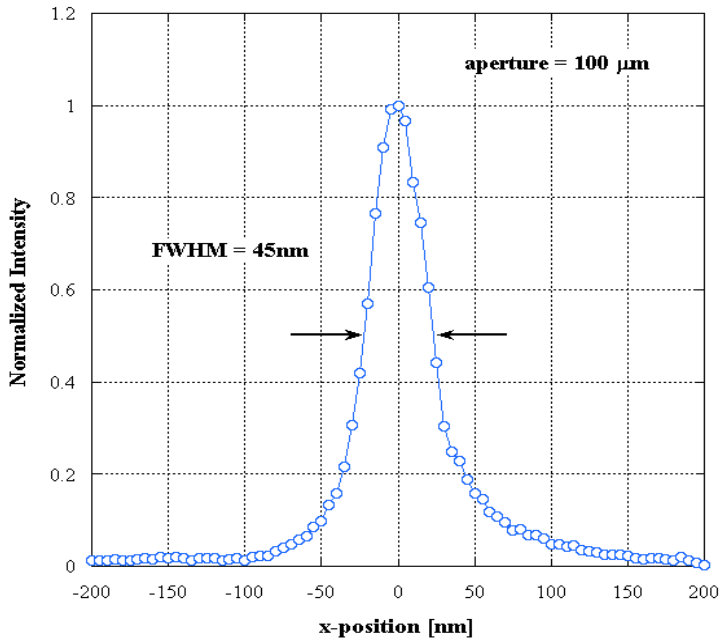


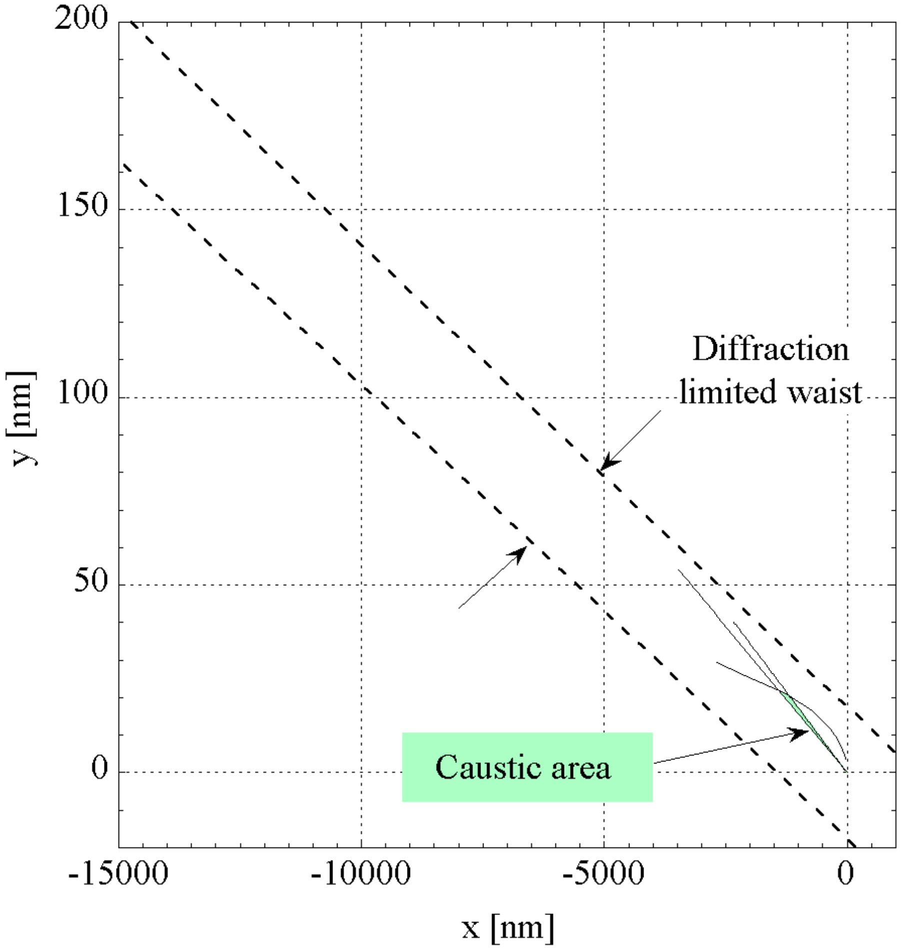


Comparison with experiments

Aberration versus diffraction

- Measured spot = 45 nm
- Diffraction limit = 35 nm
- Caustic hidden in diffraction waist





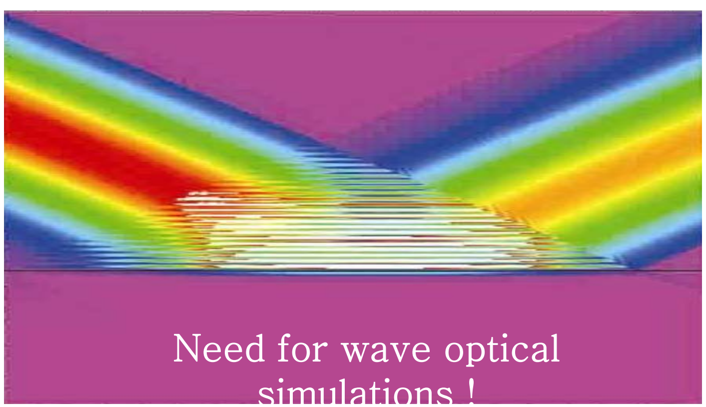
Comparison with experiments

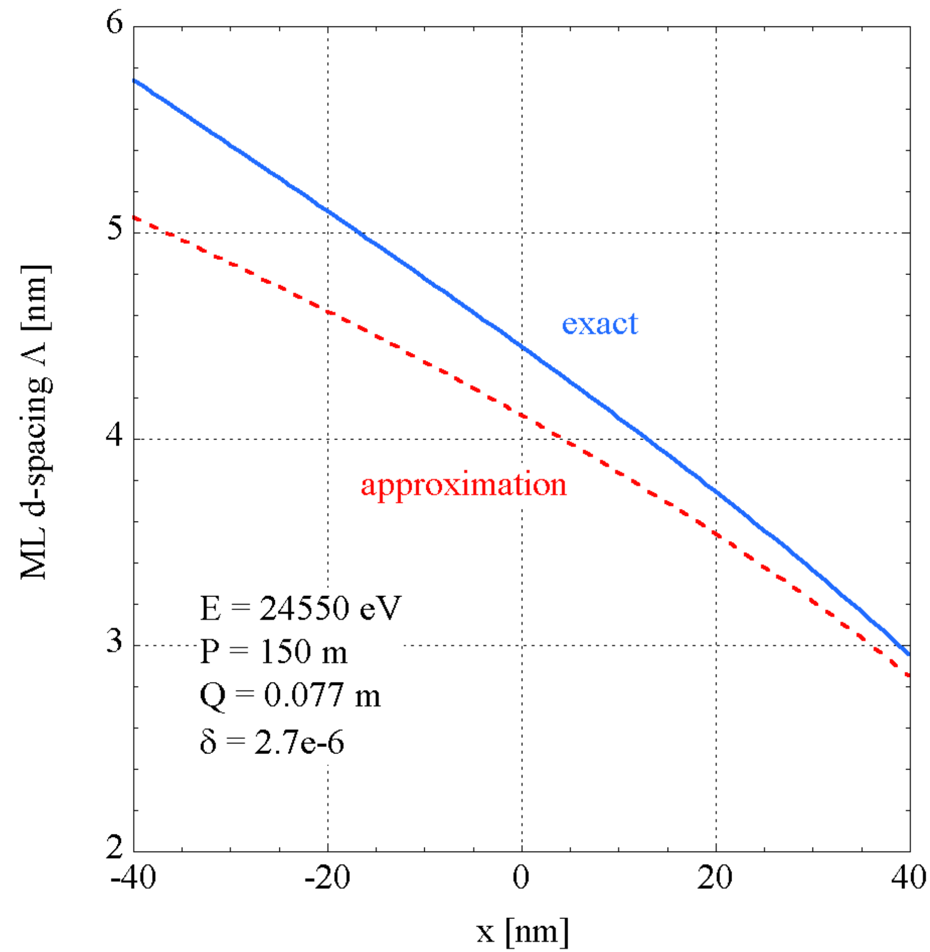
Are we already doing better ?

- ML design via corrected Bragg law

$$\Lambda = \frac{\lambda}{2\sqrt{n^2 - \cos^2\theta}} \left(\approx \frac{\lambda}{2 \cdot \sin\theta} \right)$$

- Refraction implicitly considered
- ML interface shapes not elliptic (except for surface layer)
- Aberrations reduced/suppressed ?





Summary – Perspectives

Summary

- Analytical model offers general insight into focusing performance of curved multilayers
- nm aberrations in the focal plane
- µm variations of the focal length
- Reduced aberrations for larger angles of incidence
- Chromatic aberrations negligible
- Comparative results obtained by exact ray tracing

Perspectives

- Information on intensity distribution
- Full wave optical treatment of curved multilayers → PhD project (ESRF/Univ.Göttingen)
- Improved experimental results