Modelling wave propagation across arbitrary surfaces

David Laundy
Stephen Higgins
Marion Bowler
Daresbury Laboratory, UK

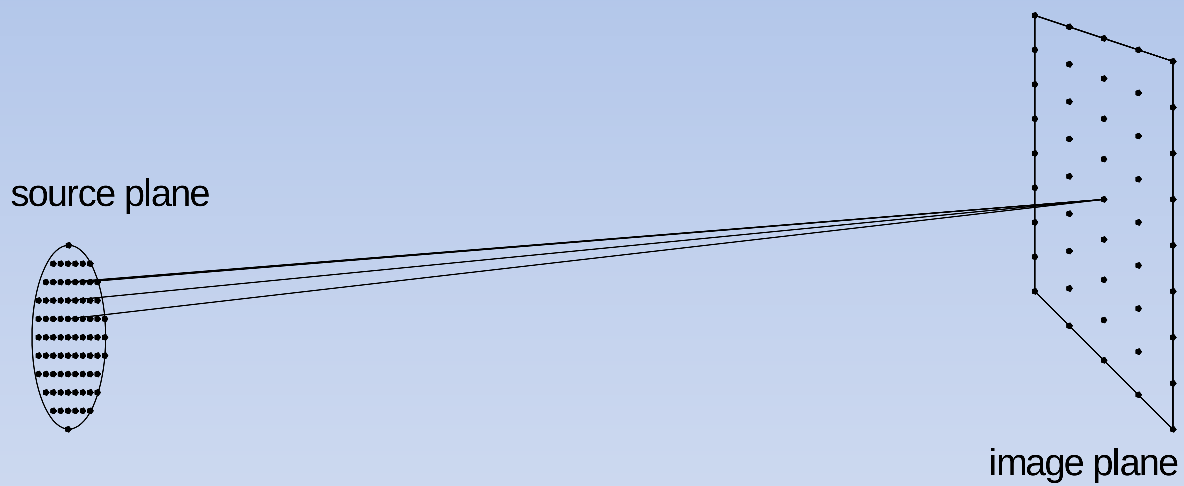
Helmholtz equation – Maxwell's equations for electric field at angular frequency ω.

$$\nabla^2 \mathbf{E} + k^2 \mathbf{E} = 0 \qquad k = \omega / c$$

Need to use the wave equation when the source is coherent e.g. FEL source. For incoherent source ray tracing can be used.
For a FEL, micro-bunching ensures that electrons emit radiation in phase with each other.
For an incoherent source the emission has random phases.
For coherent source, need to know the radiation phase at a plane defining the source e.g. for full coherence, take the phase as constant.

Free space propagation equation

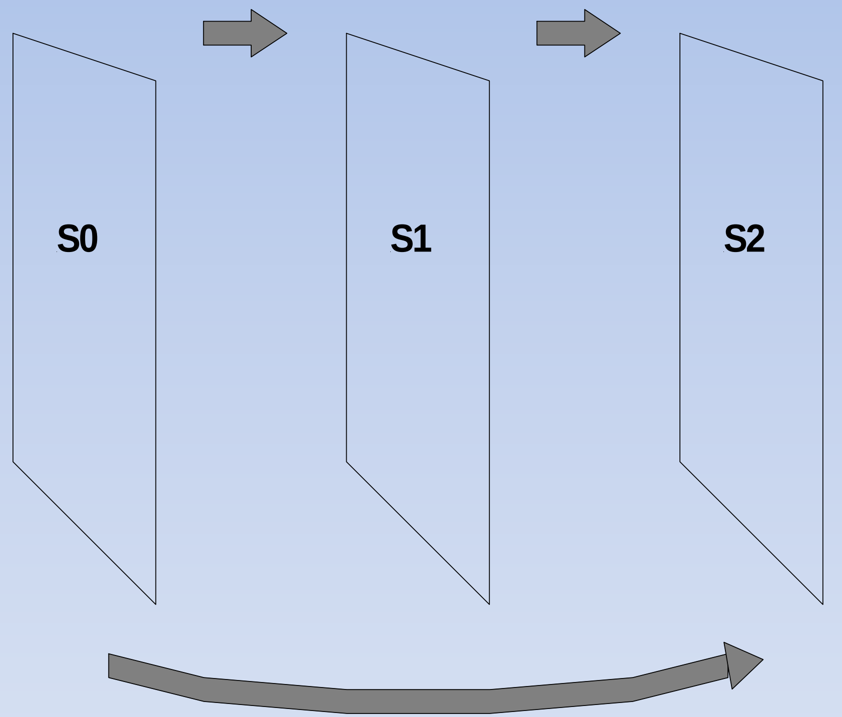
$$\mathbf{E}(\mathbf{r}) = \frac{i}{\lambda} \iint_{S_0} \mathbf{E}_0(\mathbf{r}) \frac{\exp[-ik\rho(\mathbf{r}-\mathbf{r}_0)]}{\rho(\mathbf{r}-\mathbf{r}_0)} \cos\theta \, dS_0 \qquad k = 2\pi/\lambda$$



Field is known on the source plane
Calculate field on image plane using free space propagation

Cascaded propagation

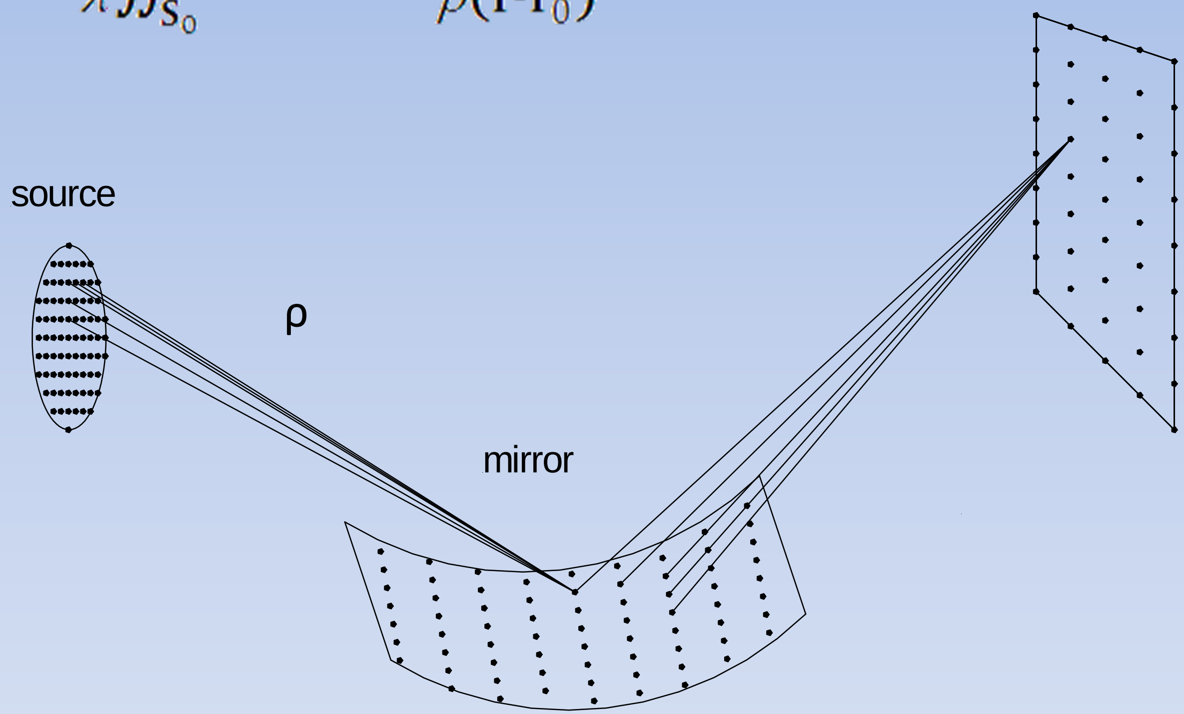
$$\mathbf{E}(\mathbf{S}_1) = \iint \vec{\mathrm{K}}(\mathbf{r}_1, \mathbf{r}_0)\mathbf{E}(\mathbf{S}_0)\mathrm{d}\mathbf{S}_0 \qquad \mathbf{E}(\mathbf{S}_2) = \iint \vec{\mathrm{K}}(\mathbf{r}_2, \mathbf{r}_1)\mathbf{E}(\mathbf{S}_1)\mathrm{d}\mathbf{S}_1$$



$$\mathbf{E}(\mathbf{S}_2) = \iint \vec{\mathrm{K}}(\mathbf{r}_2, \mathbf{r}_0)\mathbf{E}(\mathbf{S}_0)\mathrm{d}\mathbf{S}_0$$

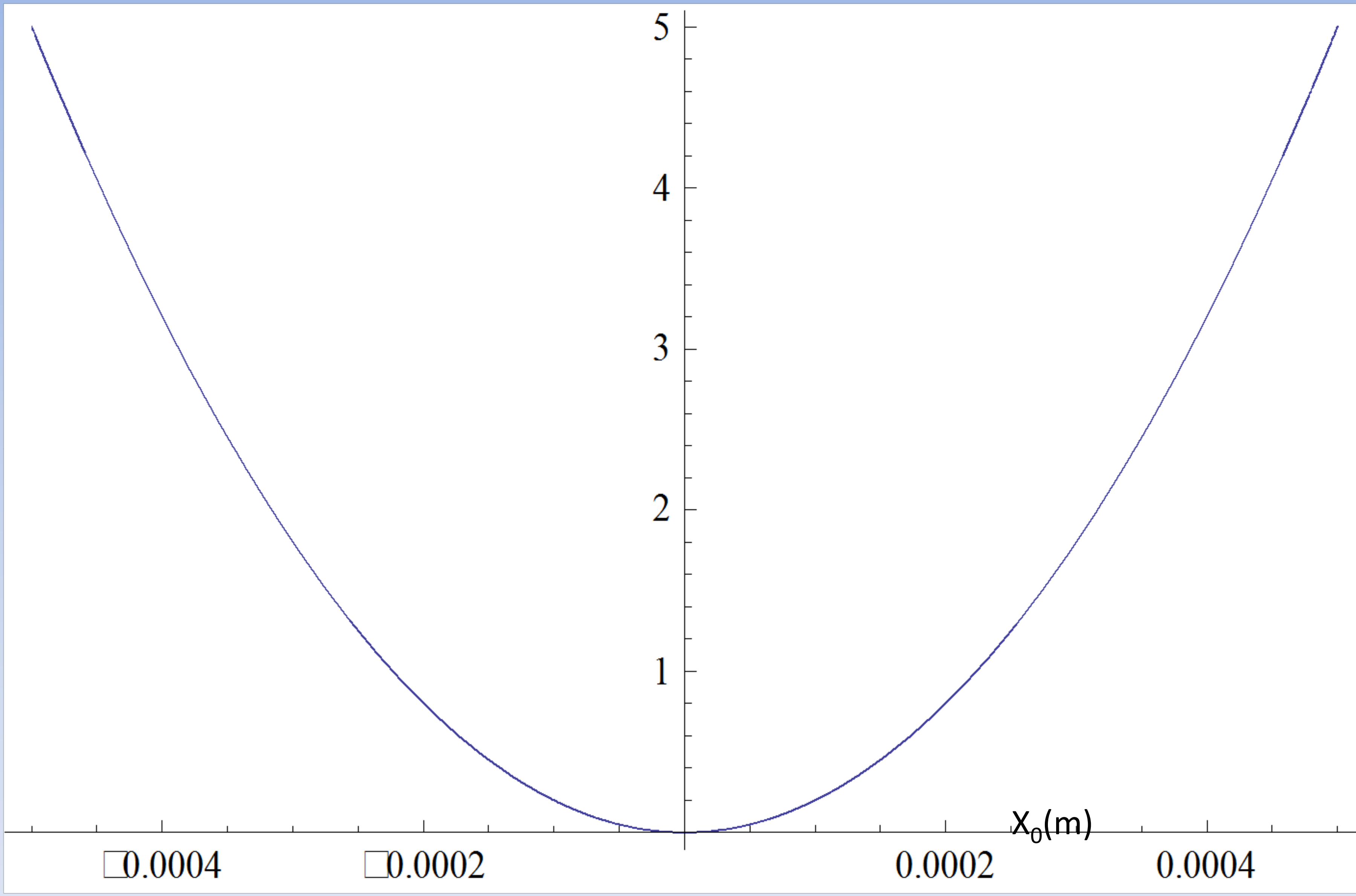
Free space propagation equation

$$\mathbf{E}(\mathbf{r}) = \frac{i}{\lambda} \iint_{S_0} \mathbf{E}_0(\mathbf{r}) \frac{\exp[-ik\rho(\mathbf{r}-\mathbf{r}_0)]}{\rho(\mathbf{r}-\mathbf{r}_0)} \cos\theta \, dS_0 \qquad k = 2\pi/\lambda$$



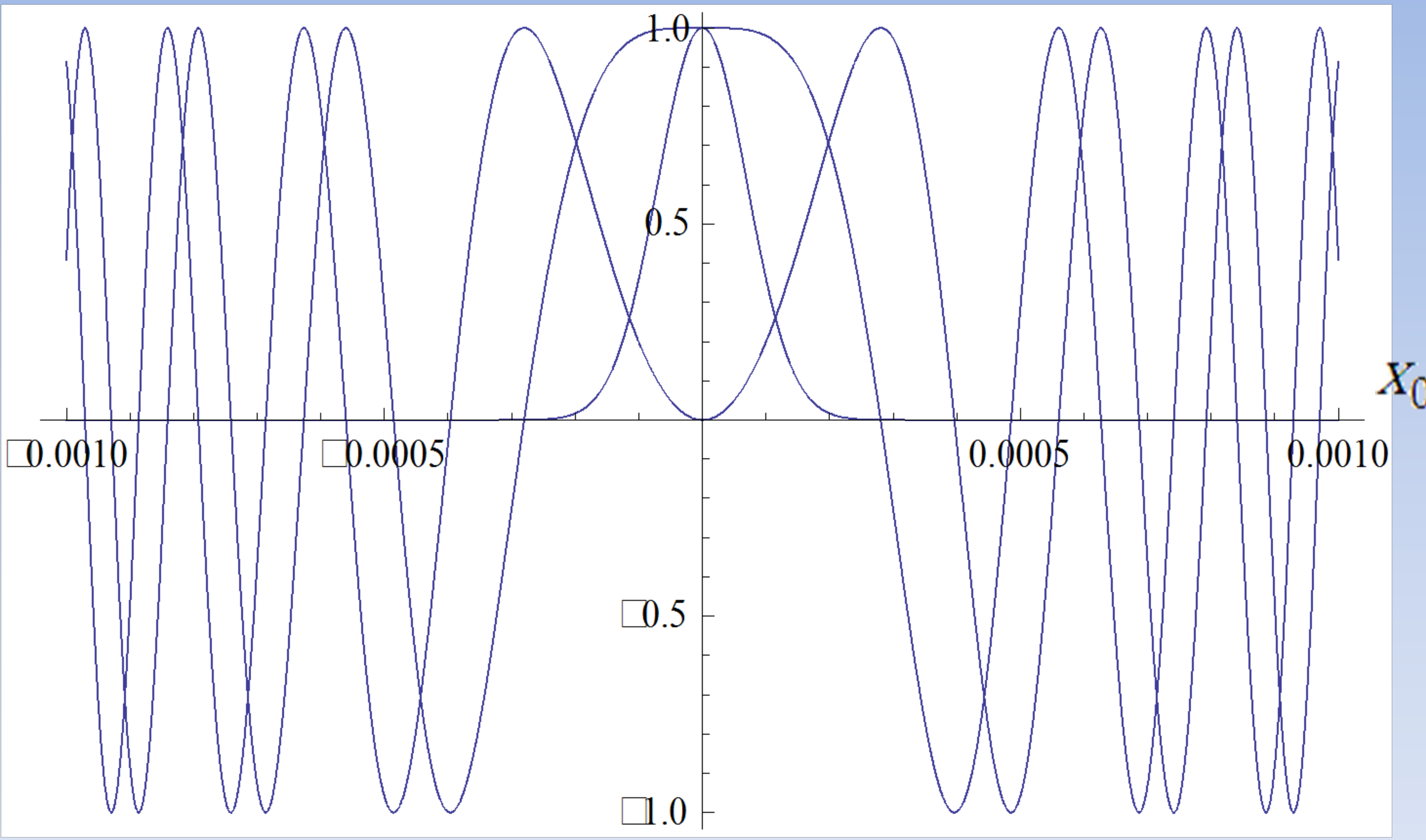
To propagate over a mirror, first free space propagate onto the surface of the mirror.
Then propagate from the surface to the image plane.
FOCUS code S Higgins, M Bowler.

Path length change ρ/λ

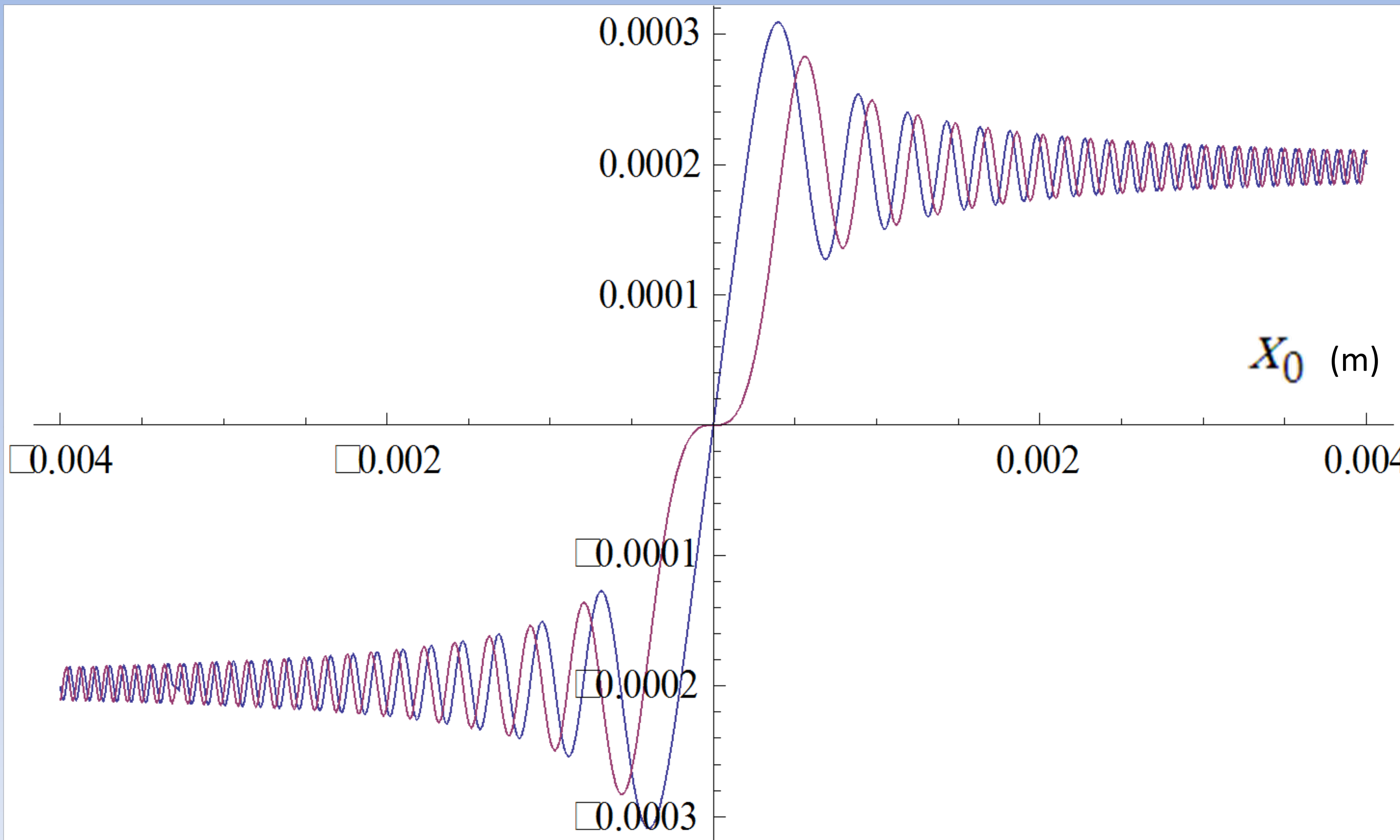


$$\mathbf{E}(\mathbf{r})=\frac{i}{\lambda}\iint_{S_0} \mathbf{E}_0(\mathbf{r})\frac{\exp[-ik\rho(\mathbf{r}-\mathbf{r}_0)]}{\rho(\mathbf{r}-\mathbf{r}_0)}\cos\theta\, dS_0$$

$$\frac{\exp[-ik\rho(x-x_0)]}{\rho(x-x_0)}$$



$$\int \frac{\exp[-ik\rho(x-x_0)]}{\rho(x-x_0)} \mathrm{d}x_0$$

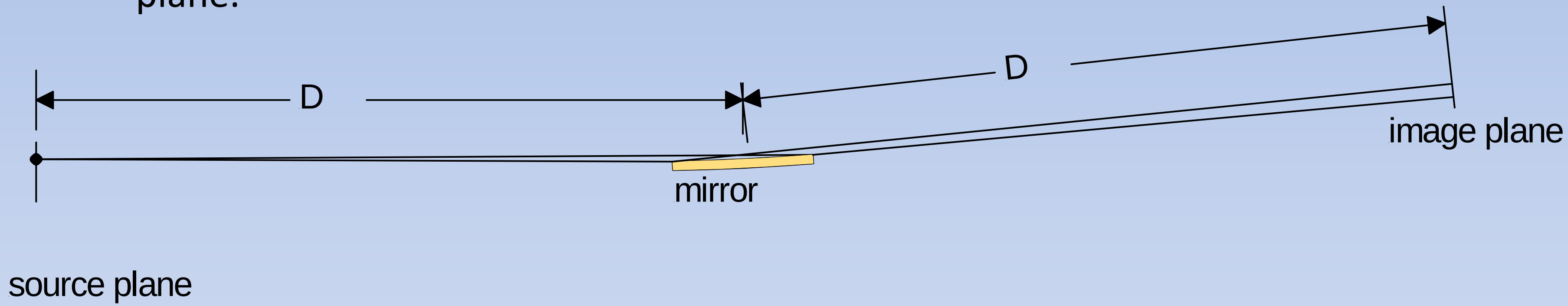


Example:

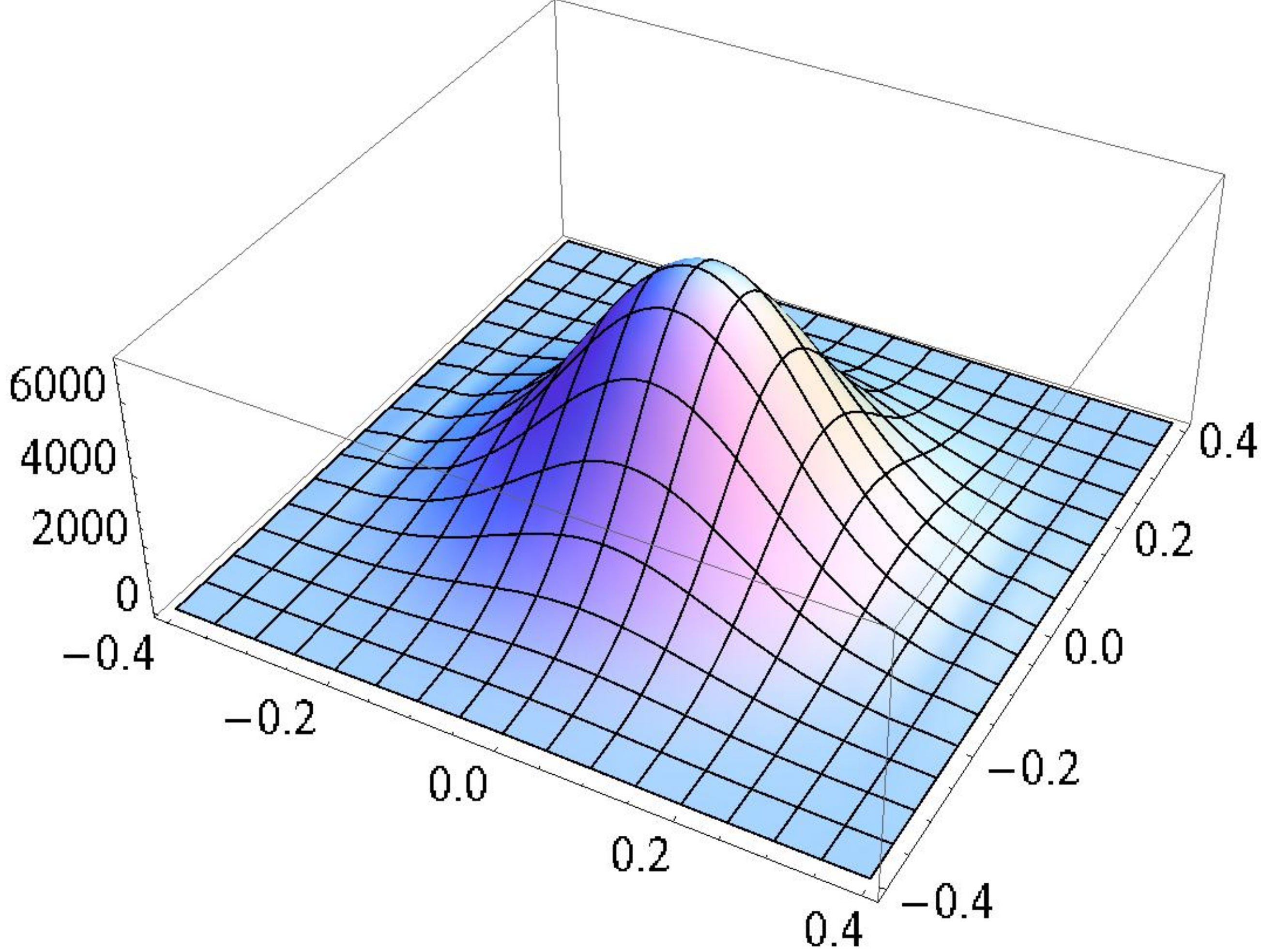
FEL source. Source size 70µm× 70µm Gaussian. Source is a plane with a grid of 2500 points.

Optical element is a plane mirror with a random surface profile of 2nm peak to peak. There are 4000 points on the mirror surface.

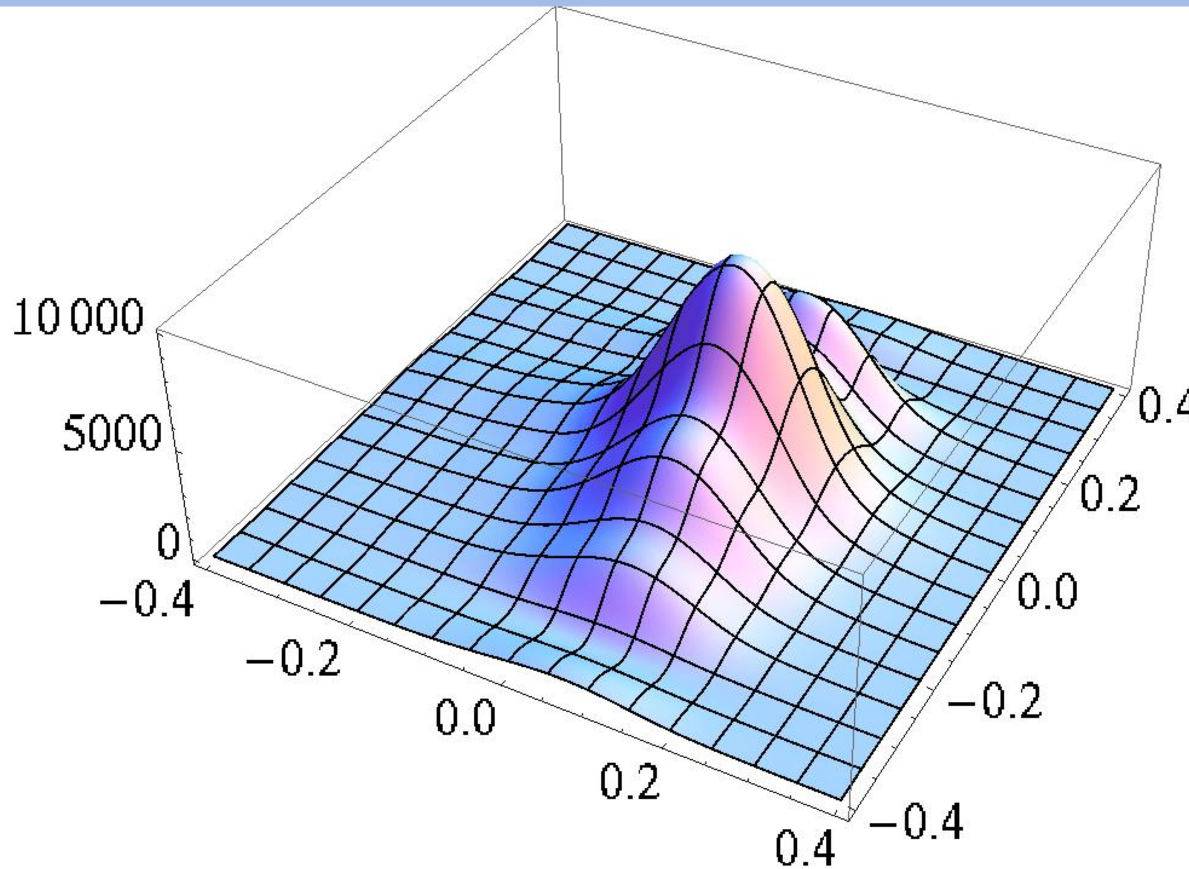
Image plane is 500m from the mirror. There are 40000 points on the image plane.



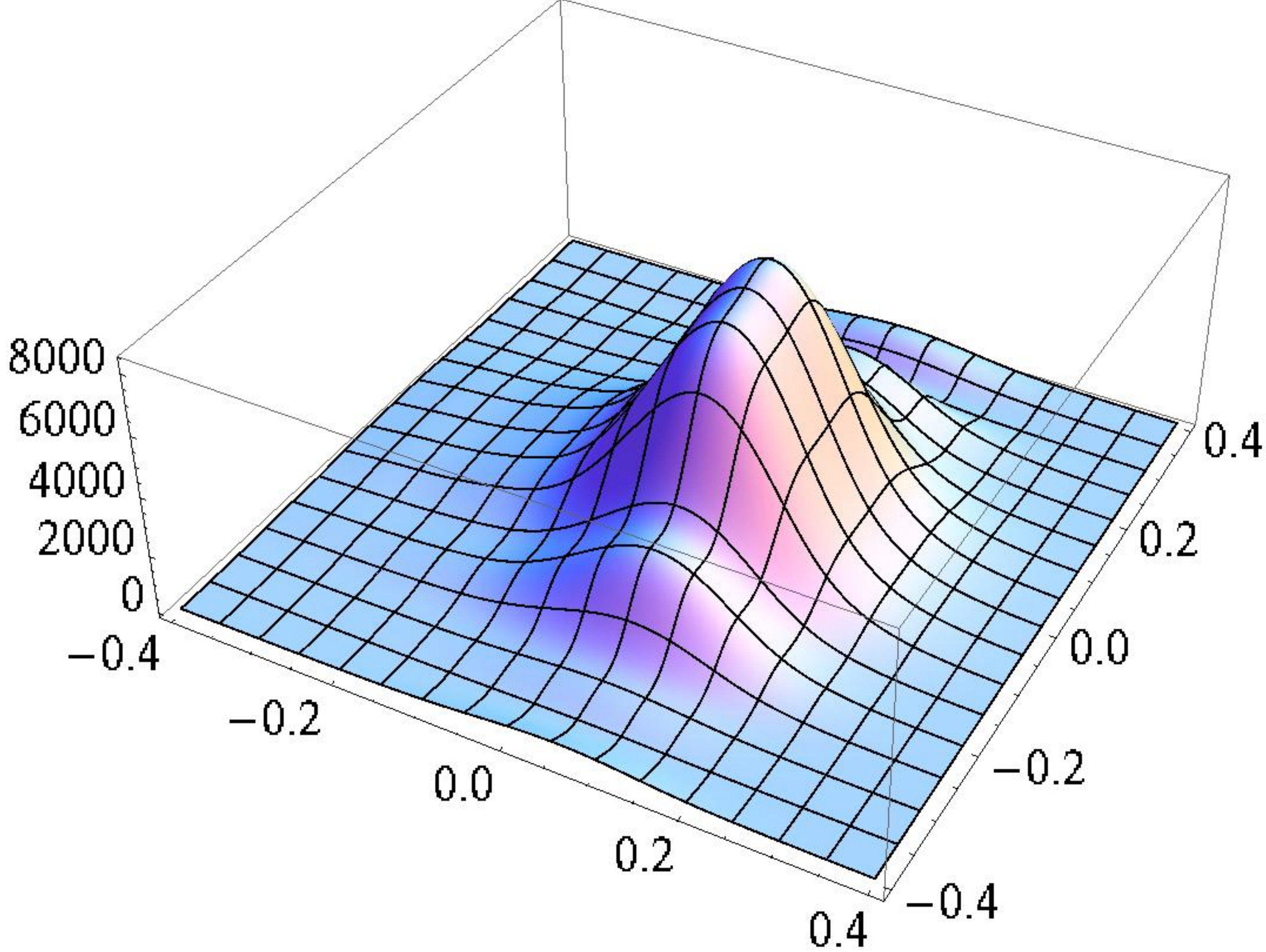
Source is fully coherent and is modelled as a having constant phase over the source plane. Optical element is a plane mirror at 500m. Image plane is 500m further downstream.



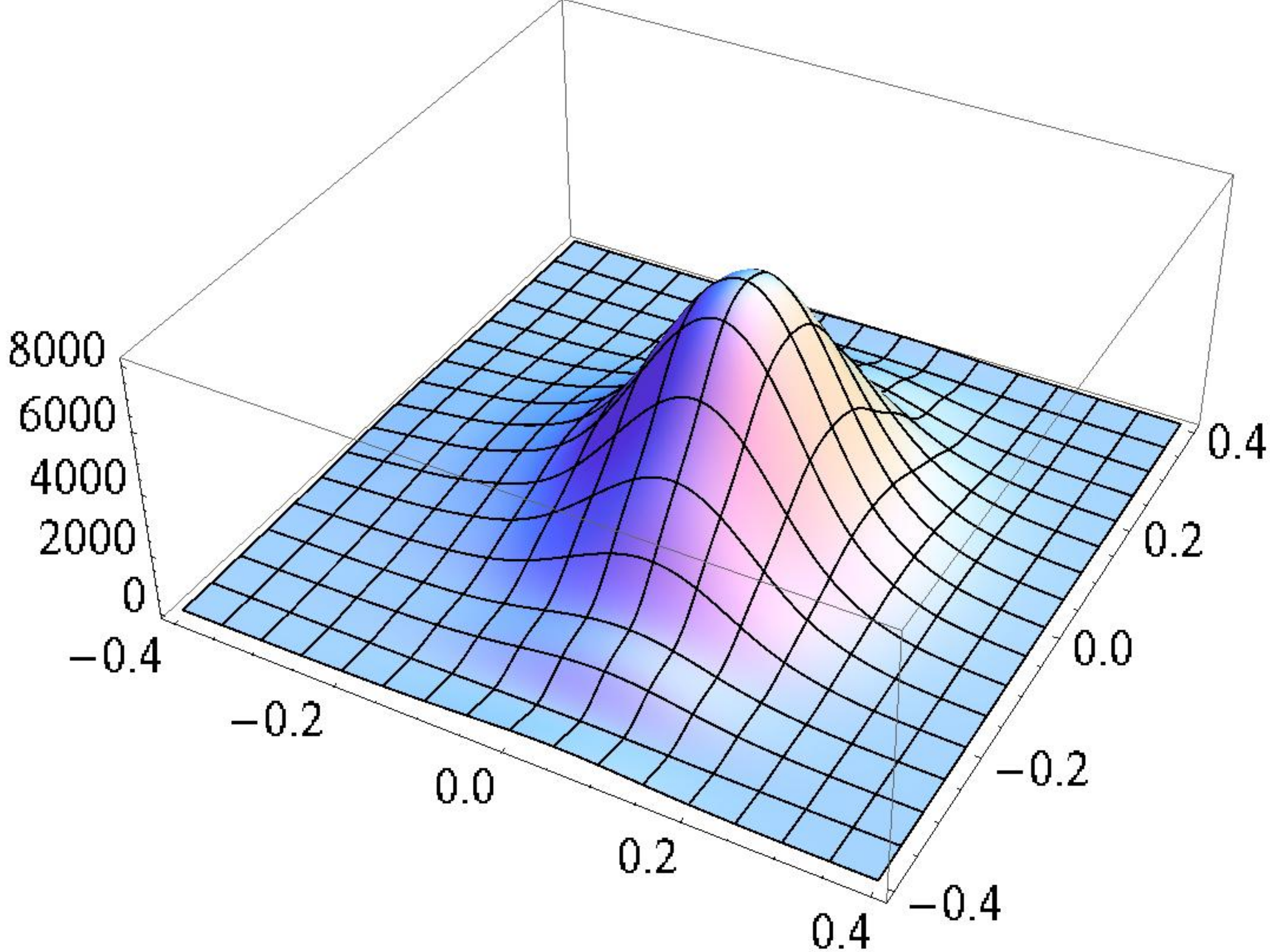
Intensity distribution at the image plane.
Flat mirror
Mirror glancing angle = 0.4 degrees.



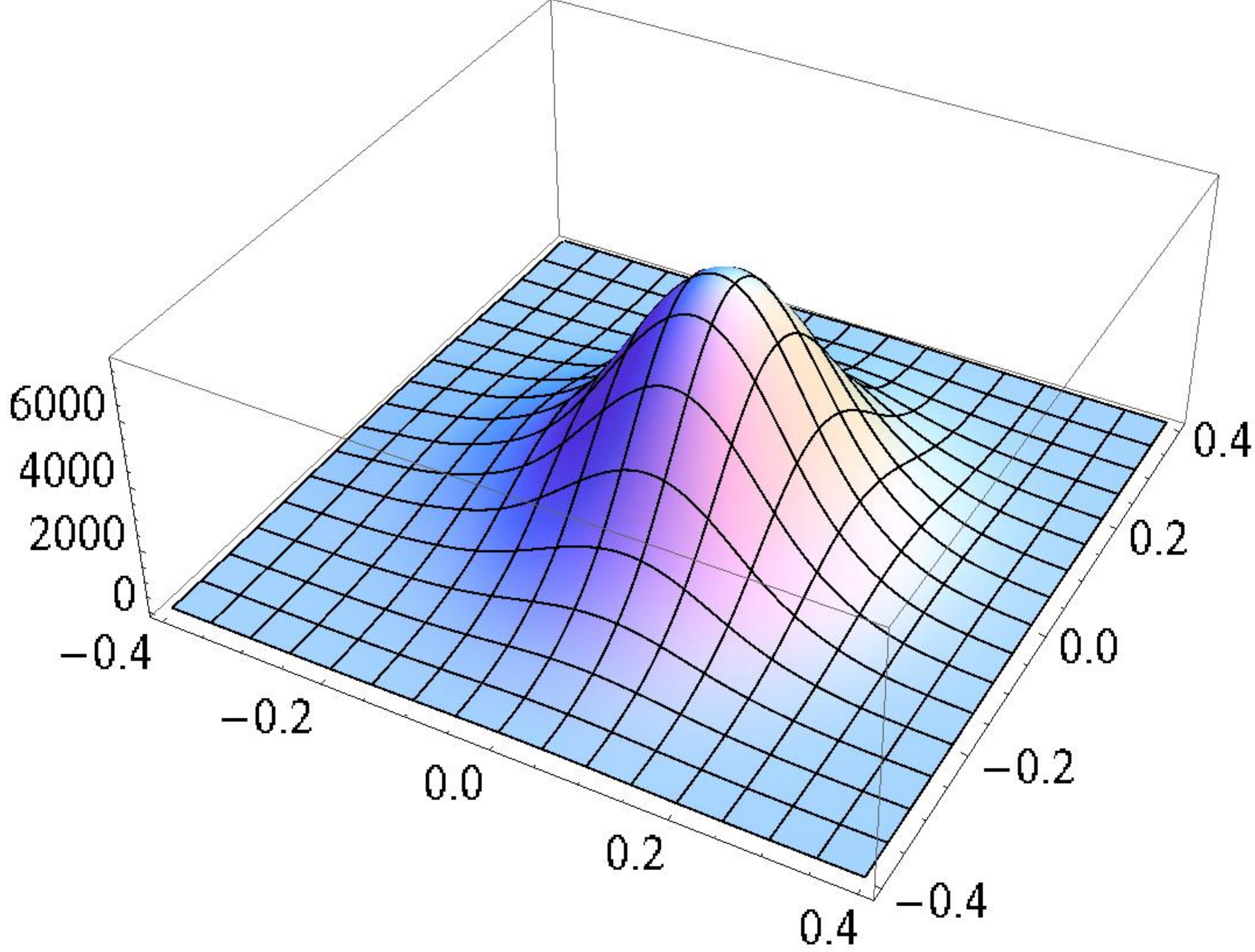
Intensity distribution at the image plane.
Mirror glancing angle = 0.4 degrees.



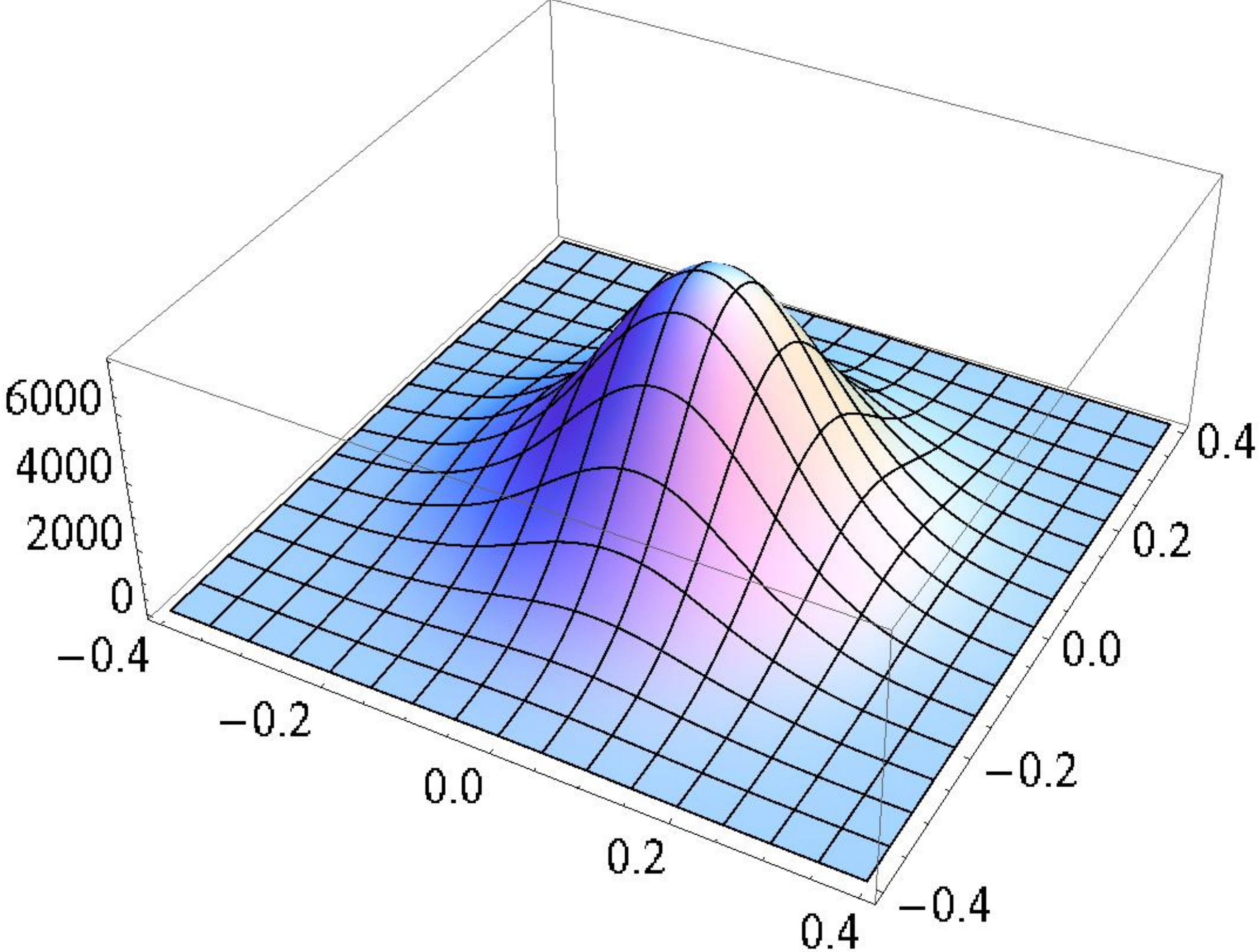
Intensity distribution at the image plane.
Mirror glancing angle = 0.3 degrees.



Intensity distribution at the image plane.
Mirror glancing angle = 0.2 degrees.

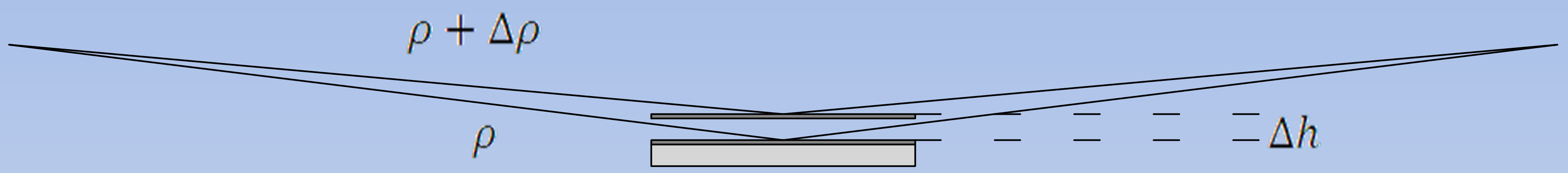


Intensity distribution at the image plane.
Mirror glancing angle = 0.1 degrees.



Intensity distribution at the image plane.
Mirror glancing angle = 0.05 degrees.

Intereference reduces as the mirror angle reduces.



Explanation

Path length depends on height displacement-

$$\Delta\rho = -2\theta\Delta h$$

Where ϑ is mirror incidence angle (relative to the surface). Hence contribution of height changes is reduced by a factor ϑ which is of order 10^{-3}

Future

More information on coherence.