# RADIA

## a 3D Magnetostatics Computer Code

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# Motivation

- Fast computation of stationary Magnetic Field produced by Permanent Magnets, Coils and Iron Blocks and in 3D space
- Optimized for the design of Accelerator Magnets, Undulators and Wigglers:
  - high accuracy of the Field outside magnet blocks
  - fast computation of Field Integrals and Particle Trajectories
- Simple Interface with Scripting and Visualization, allowing fast set-up of complicated 3D geometries

# Previous Codes

- **GFUN** (Trowbridge, Rutherford Laboratory, 1970s)
  - "volume (/magnetization) integral" approach: proof of principle
  - written in FORTRAN
- **Radia-1, B3D** (J.Chavanne, P.Elleaume, ESRF, 1988-95)
  - computation of field and field integrals produced by uniformly magnetized rectangular parallelepipeds (permanent magnets)
  - relaxation; linear and non-linear magnetic materials; support of symmetries
  - written in C; interfaced to Wingz

# Method

## Magnetic Field created by Uniformly Magnetized Volumes

Poisson equation for scalar magnetic potential:

$$\nabla \mathbf{B} = 0,$$
$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}),$$
$$\mathbf{H} = -\nabla \varphi,$$
$$\Delta \varphi = \nabla \mathbf{M},$$

Solution through volume and surface integrals:

$$\varphi(\mathbf{r}) = \frac{-1}{4\pi} \iiint_{V'} \frac{\nabla \mathbf{M}}{|\mathbf{r}' - \mathbf{r}|} dV' + \frac{1}{4\pi} \oiint_{S'} \frac{\mathbf{M}\,\mathbf{n}_{S'}}{|\mathbf{r}' - \mathbf{r}|} dS',$$
$$\mathbf{H}(\mathbf{r}) = \frac{1}{4\pi} \iiint_{V'} \frac{(\mathbf{r}' - \mathbf{r})\,\nabla \mathbf{M}}{|\mathbf{r}' - \mathbf{r}|^3} dV' - \frac{1}{4\pi} \oiint_{S'} \frac{(\mathbf{r}' - \mathbf{r})\,\mathbf{M}\,\mathbf{n}_{S'}}{|\mathbf{r}' - \mathbf{r}|^3} dS',$$

Magnetic field created by uniformly magnetized volume:

$$\text{Si } \mathbf{M} = \mathbf{const}:$$
$$\mathbf{H}(\mathbf{r}) = \mathbf{Q}(\mathbf{r})\,\mathbf{M},$$
$$\mathbf{Q}(\mathbf{r}) = \frac{1}{4\pi} \oiint_{S'} \frac{(\mathbf{r} - \mathbf{r}') \otimes \mathbf{n}_{S'}}{|\mathbf{r} - \mathbf{r}'|^3} dS'; \qquad (\mathbf{a} \otimes \mathbf{b})\mathbf{c} \equiv \mathbf{a}(\mathbf{bc})$$

Field integral along straight line:

$$\mathbf{I}(\mathbf{r}_0, \mathbf{v}) \equiv \int_{-\infty}^{+\infty} \mathbf{H}(\mathbf{r}_0 + \mathbf{v}s)\,ds = \mathbf{G}(\mathbf{r}_0, \mathbf{v})\,\mathbf{M}, \qquad |\mathbf{v}| = 1,$$
$$\mathbf{G}(\mathbf{r}_0, \mathbf{v}) = \frac{1}{2\pi} \oiint_{S'} \frac{[\mathbf{v} \times [(\mathbf{r}_0 - \mathbf{r}') \times \mathbf{v}]] \otimes \mathbf{n}_{S'}}{|(\mathbf{r}_0 - \mathbf{r}') \times \mathbf{v}|^2} dS'$$

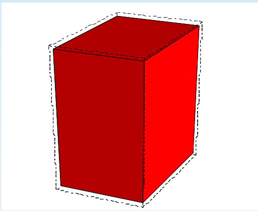
# Method

## Uniformly Magnetized Rectangular Parallelepiped

Magnetic field: $\mathbf{H} = \mathbf{Q}\,\mathbf{M}$

Field integral along straight line:

$$\mathbf{I}(\mathbf{r}_0,\mathbf{v}) = \mathbf{G}(\mathbf{r}_0,\mathbf{v})\,\mathbf{M}$$



$(x_c, y_c, z_c)$ - coordinates of the block center

$(w_x, w_y, w_z)$ - block dimensions

$(x, y, z)$ - coordinates of an observation point / point on integration line

$(v_x, v_y, v_z)$ - unit vector parallel to the integration line

$$Q_{xx} = \frac{1}{4\pi}\sum_{i,j,k=1}^{2}(-1)^{i+j+k+1}\tan^{-1}[x_i^{-1} y_j z_k (x_i^2+y_j^2+z_k^2)^{-1/2}],$$

$$Q_{xy} = \frac{1}{4\pi}\ln\left[\prod_{i,j,k=1}^{2}[z_k+(x_i^2+y_j^2+z_k^2)^{1/2}]^{(-1)^{i+j+k}}\right],$$

$$Q_{ll'} = Q_{l'l},\quad l,l' = x,y,z,$$

$$x_{1,2} = x_c - x \mp w_x/2,\quad y_{1,2} = y_c - y \mp w_y/2,\quad z_{1,2} = z_c - z \mp w_z/2,$$

$$G_{xx} = \frac{1}{2\pi}\sum_{i,j,k=1}^{2}(-1)^{i+j+k}\Bigg[(v_z z_k u_y^{-1} - v_y y_j u_z^{-1})\tan^{-1}\left[\frac{u_x x_i - (v_y y_j + v_z z_k)v_x}{v_z y_j - v_y z_k}\right] +$$
$$+ v_x x_i\left[u_y^{-1}\tan^{-1}\left[\frac{u_y y_j - (v_x x_i + v_z z_k)v_y}{v_x z_k - v_z x_i}\right] + u_z^{-1}\tan^{-1}\left[\frac{u_z z_k - (v_x x_i + v_y y_j)v_z}{v_x y_j - v_y x_i}\right]\right] +$$
$$+ [(v_x y_j - v_y x_i)v_z u_z^{-1} + (v_x z_k - v_z x_i)v_y u_y^{-1}]\,L_{ijk}\Bigg],$$

$$G_{xy} = \frac{1}{2\pi}\sum_{i,j,k=1}^{2}(-1)^{i+j+k}\Bigg[(v_y x_i - v_x y_j)u_z^{-1}\tan^{-1}\left[\frac{u_z z_k - (v_x x_i + v_y y_j)v_z}{v_x y_j - v_y x_i}\right] +$$
$$+ [(v_x x_i + v_y y_j)v_z u_z^{-1} - z_k]\,L_{ijk}\Bigg],$$

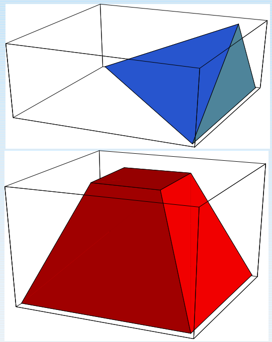
$$L_{ijk} = \ln[(v_x y_j - v_y x_i)^2 + (v_x z_k - v_z x_i)^2 + (v_z y_j - v_y z_k)^2]/2,$$

$$u_l = 1 - v_l^2,\quad l = x,y,z$$

# Method

## Uniformly Magnetized Polyhedron

Magnetic field: $\mathbf{H} = \mathbf{Q}\,\mathbf{M}$



$(\tilde{x}_{\sigma_s}, \tilde{y}_{\sigma_s}, \tilde{z}_{\sigma_s})$ , $s = 1, 2, \ldots, N_\sigma$ ,

- coord. of vertex points of the face $\sigma$

$\mathbf{n}_\sigma$ - external normal to the face $\sigma$

$N_f$ - number of faces

$(x_0, y_0, z_0)$ – coord. of observation point

$$\mathbf{Q} = \sum_{\sigma=1}^{N_f} \mathbf{T}_\sigma^{-1} (\mathbf{F}_\sigma \otimes \mathbf{k}) \mathbf{T}_\sigma,$$

$$\mathbf{F}_\sigma = \frac{1}{4\pi} \sum_{s=1}^{N_\sigma} \begin{bmatrix} f_x(x_{\sigma_s}, a_{\sigma_s}, b_{\sigma_s}, z_\sigma) - f_x(x_{\sigma_{s+1}}, a_{\sigma_s}, b_{\sigma_s}, z_\sigma) \\ f_y(x_{\sigma_s}, a_{\sigma_s}, b_{\sigma_s}, z_\sigma) - f_y(x_{\sigma_{s+1}}, a_{\sigma_s}, b_{\sigma_s}, z_\sigma) \\ f_z(x_{\sigma_s}, a_{\sigma_s}, b_{\sigma_s}, z_\sigma) - f_z(x_{\sigma_{s+1}}, a_{\sigma_s}, b_{\sigma_s}, z_\sigma) + \varphi(x_{\sigma_s}, x_{\sigma_{s+1}}, a_{\sigma_s}, b_{\sigma_s}, z_\sigma) \end{bmatrix},$$

$$a_{\sigma_s} = (y_{\sigma_{s+1}} - y_{\sigma_s}) / (x_{\sigma_{s+1}} - x_{\sigma_s}),$$

$$b_{\sigma_s} = y_{\sigma_s} - a_{\sigma_s} x_{\sigma_s},$$

$$\begin{bmatrix} x_{\sigma_s} \\ y_{\sigma_s} \\ z_\sigma \end{bmatrix} = \mathbf{T}_\sigma \begin{bmatrix} \tilde{x}_{\sigma_s} - x_0 \\ \tilde{y}_{\sigma_s} - y_0 \\ \tilde{z}_{\sigma_s} - z_0 \end{bmatrix}; \quad \mathbf{T}_\sigma = \begin{bmatrix} n_{y_\sigma}^2 (n_{z\sigma} + 1)^{-1} + n_{z\sigma} & -n_{x\sigma} n_{y_\sigma} (n_{z\sigma} + 1)^{-1} & -n_{x\sigma} \\ -n_{x\sigma} n_{y_\sigma} (n_{z\sigma} + 1)^{-1} & n_{x\sigma}^2 (n_{z\sigma} + 1)^{-1} + n_{z\sigma} & -n_{y_\sigma} \\ n_{x\sigma} & n_{y_\sigma} & n_{z\sigma} \end{bmatrix}, \ \mathbf{n}_\sigma \neq -\mathbf{k}; \quad \mathbf{T}_\sigma = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \ \mathbf{n}_\sigma = -\mathbf{k},$$

$$\tilde{x}_{\sigma N_\sigma + 1} \equiv \tilde{x}_{\sigma 1}, \ \tilde{y}_{\sigma N_\sigma + 1} \equiv \tilde{y}_{\sigma 1},$$

$$f_x(x, a, b, z) = -a(a^2 + 1)^{-1/2} \ln[[ab + (a^2 + 1)x] + (a^2 + 1)^{1/2} R(x, a, b, z)] + \ln[ax + b + R(x, a, b, z)],$$

$$f_y(x, a, b, z) = (a^2 + 1)^{-1/2} \ln[[ab + (a^2 + 1)x] + (a^2 + 1)^{1/2} R(x, a, b, z)],$$

$$f_z(x, a, b, z) = \tan^{-1}[q_1(x, a, b, z) / q_2(x, a, b, z)],$$

$$\varphi(x_1, x_2, a, b, z) = \pi \quad \theta[d(a, b, z)] \operatorname{sgn}(x_2 - x_1) \times$$

$$\times \sum_{i=1}^{2} \beta[x_{r_i}(a, b, z), a, b, z] \theta[[x_1 - x_{r_i}(a, b, z)][x_{r_i}(a, b, z) - x_2]] \times$$

$$\times \operatorname{sgn}[q_1[x_{r_i}(a, b, z), a, b, z] q_2'[x_{r_i}(a, b, z), a, b, z]],$$

$$q_1(x, a, b, z) = z^{-1}[2abz^2(x^2 + z^2) + (az^2 + bx)(a^2 z^2 + b^2)[ax + b + R(x, a, b, z)]],$$

$$q_2(x, a, b, z) = (a^2 z^2 - b^2)(x^2 + z^2) + (ax - b)(a^2 z^2 + b^2)[ax + b + R(x, a, b, z)],$$

$$q_2'(x, a, b, z) = 2[(a^2 - 1)b^2 + (a^2 + 1)a^2 z^2]x + (a^2 z^2 + b^2)[az^2 + 2a(a^2 + 1)x^2 + (2a^2 - 1)bx] / R(x, a, b, z),$$

$$\beta(x, a, b, z) = \theta[\operatorname{sgn}[[(a^2 z^2 - b^2)(x^2 + z^2) + (a^2 x^2 - b^2)(a^2 z^2 + b^2)](b - ax)]],$$

$$d(a, b, z) = [(a^2 + 1)z^2 + b^2][4a^2 b^2 (a^2 z^2 + b^2) - (a^2 z^2 - b^2)^2],$$

$$x_{r1,2}(a, b, z) = [ab(a^2 z^2 + b^2)^2 \pm (a^2 z^2 - b^2)[d(a, b, z)]^{1/2}] / [4a^2 b^4 - (a^2 + 1)(a^2 z^2 - b^2)^2],$$

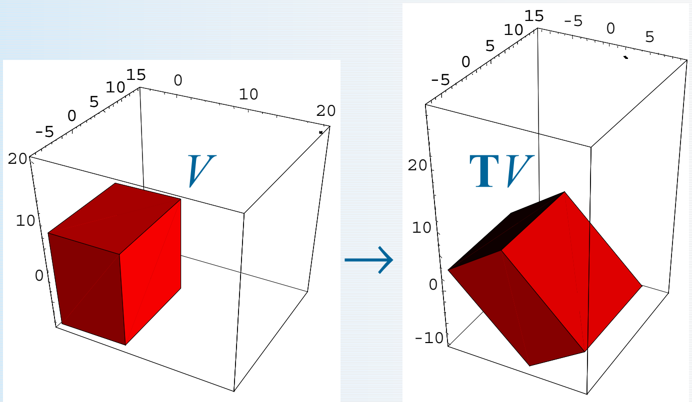
$$R(x, a, b, z) = [x^2 + (ax + b)^2 + z^2]^{1/2}$$

# Method

## Space Transformations and Symmetries

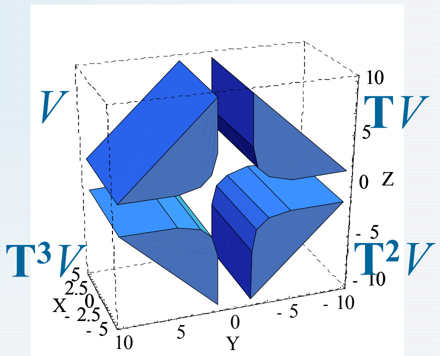
- Transformations

$$\mathbf{H}(\mathbf{r},\mathbf{T}V)=\mathbf{T}\mathbf{H}(\mathbf{T}^{-1}\mathbf{r},V)$$




- Symmetries (multiplicity $m > 1$)

$$\mathbf{H}_{tot}(\mathbf{r})=\sum_{i=0}^{m-1}\mathbf{H}(\mathbf{r},\mathbf{T}^{i}V)=\sum_{i=0}^{m-1}\mathbf{T}^{i}\mathbf{H}(\mathbf{T}^{-i}\mathbf{r},V)$$




Treatment of Symmetries reduces memory requirements and speeds up computation

# Method

## Relaxation

- Interaction Matrix and Material Relations

$$\mathbf{H}_i = \sum_{k=1}^{N} \mathbf{Q}_{ik}\mathbf{M}_k + \mathbf{H}_{\mathbf{ex}\,i},$$
$$\mathbf{M}_i = \mathbf{f}_i(\mathbf{H}_i), \quad i = 1,2,\dots,N,$$

$\mathbf{H}_i$ - total field strength in the center of object $i$
$\mathbf{H}_{\mathbf{ex}\,i}$ - external field the center of the object $i$
$\mathbf{M}_k$ - magnetization in the object $k$
$\mathbf{Q}_{ik}$ - component of the Interaction Matrix (being itself a 3 x 3 matrix)
$\mathbf{f}_i(\mathbf{H})$ - magnetization vs. field strength law for the material of the object $i$

- Relaxation Scheme

$$\widetilde{\mathbf{H}}_{\mathbf{ex}\,i,p} = \mathbf{H}_{\mathbf{ex}\,i} + \sum_{k=1}^{i-1} \mathbf{Q}_{ik}\mathbf{M}_{k,p} + \sum_{k'=i+1}^{N} \mathbf{Q}_{ik'}\mathbf{M}_{k',p-1},$$
$$\mathbf{H}_{i,p} = [\mathbf{E} - \mathbf{Q}_{ii}\boldsymbol{\chi}_i(\mathbf{H}_{i,p-1})]^{-1}(\widetilde{\mathbf{H}}_{\mathbf{ex}\,i,p} + \mathbf{Q}_{ii}\mathbf{M}_{\mathbf{r}\,i}),$$
$$\mathbf{M}_{i,p} = \mathbf{f}_i(\mathbf{H}_{i,p}),$$

$\boldsymbol{\chi}_i(\mathbf{H})$ - local susceptibility tensor for the material of the object $i$
$\mathbf{M}_{\mathbf{r}\,i}$ - rem. magnetization in the object $i$

for nonlinear isotropic material:

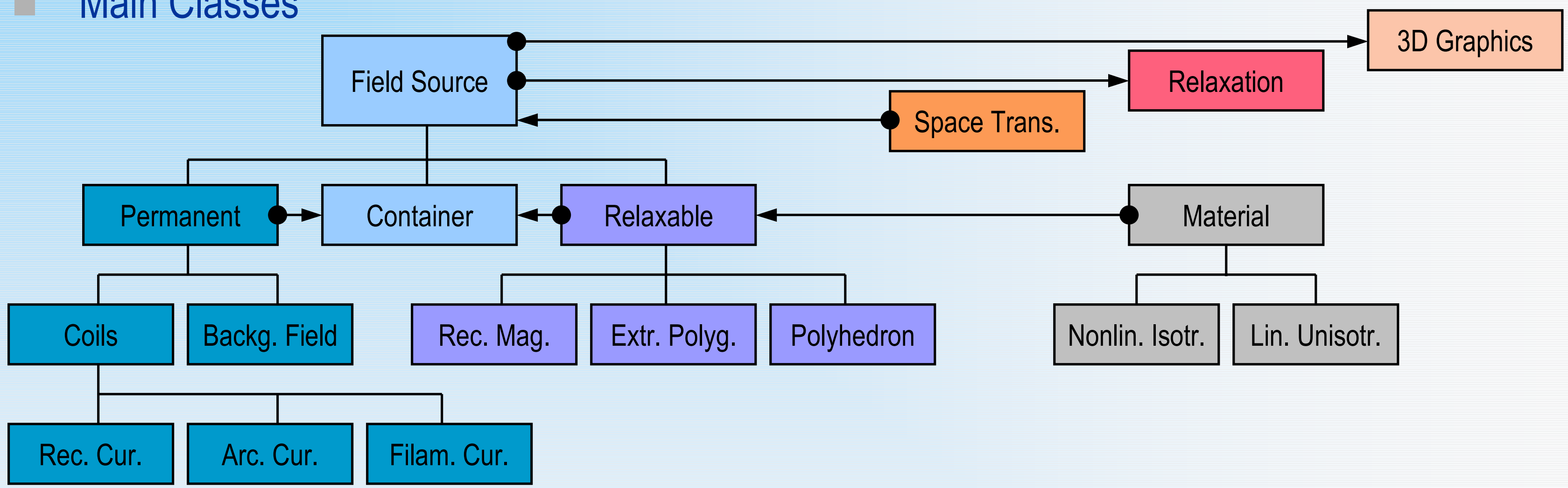
$$\boldsymbol{\chi}_i(\mathbf{H}) = \begin{cases} [f_i(|\mathbf{H}|)/|\mathbf{H}|]\,\mathbf{E}, & |\mathbf{H}| \neq 0, \\ f_i'(0)\,\mathbf{E}, & |\mathbf{H}| = 0, \end{cases}$$

No “relaxation parameter” required (!)

# Implementation

## Programmed in C++

- Main Classes




- General Methods for Field Sources
  - computation of Magnetic Field Strength, Vector Potential, Field Integral, Energy, Force, Torque
  - subdivision (segmentation)
- Radia is interfaced to Mathematica® (Wolfram Research)
- Exists on Windows, Linux, MacOS platforms
- Available for download from ESRF web site (Insertion Devices page)

# Implementation

## Comparison with a FEM Code

- **Radia** vs. commercial FEM code **FLUX3D**

  Hybrid Wiggler simulation

  Case A: Solution for 1% accuracy in **peak field**

  Case B: Solution for 10 G-cm abs. accuracy in **on-axis field integral**

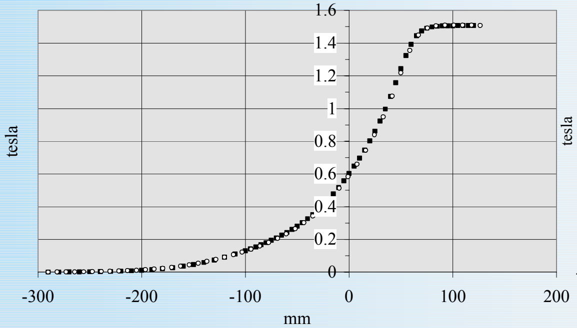
| | | **Radia** | **FLUX3D** |
|---|---|---|---|
| CPU time | A<br>B | 10 s<br>100 s | 200 s<br>∞ |
| Number of 3D elements | A<br>B | 400<br>980 | ~15 000<br>∞ |
| Memory required | A<br>B | 7 MB<br>27 MB | 11 MB<br>? |
| Accuracy of the field inside magnet blocks | | poor* | same as in the air |

* accuracy is high only in centers of 3D elements

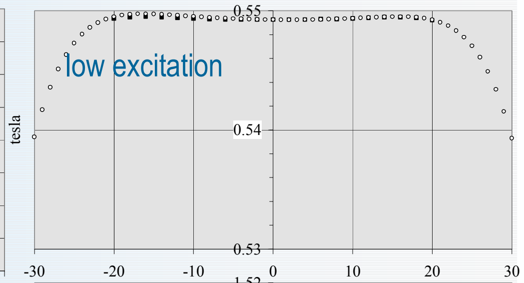
# Example of Computation

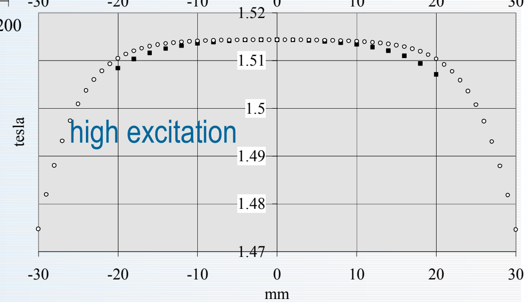
## Accelerator Magnets

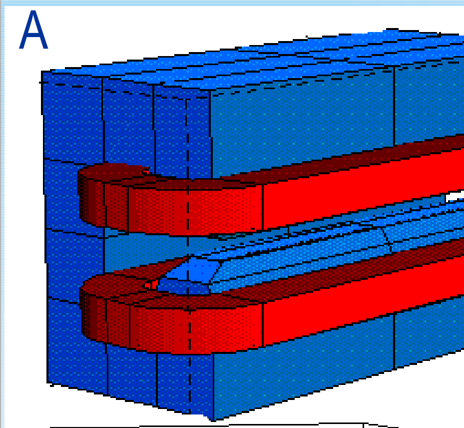
Edge Field (vs longitudinal position)

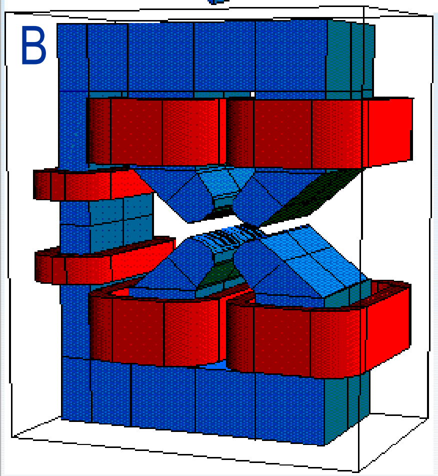



Transverse Field



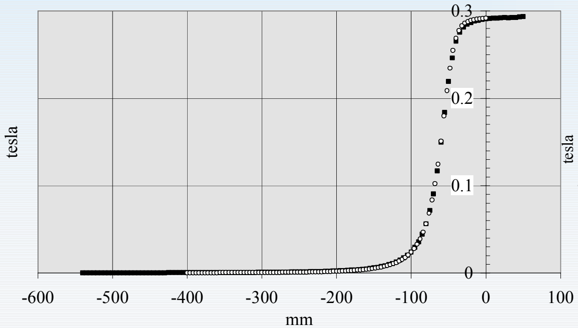



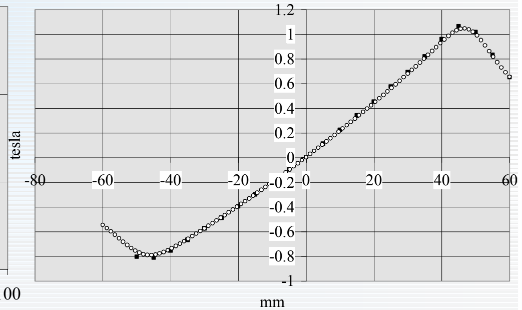

MAX-II Magnets

A: dipole

B: quadrupole with integrated sextupole component

simulation by L-J.Lindgren




Discrepancy between Radia simulations and measurements: < 1% (peak field)

# Possible Further Developments

## Direct Problems

- increasing precision inside iron blocks
- reducing memory consumption
- automatic adaptive subdivision
- automatic drivers (for solution at desired accuracy with respect to particular field components)
- electrostatics and other problems of math. physics allowing application of potentials

## Inverse Problems

- multi-parameter geometry optimization
- " $\mathbf{H} = \mathbf{Q}\,\mathbf{M} \Rightarrow \mathbf{M} = \mathbf{Q}^{-1}\mathbf{H}$ " (+ regularization)

## General Programming

- releasing Radia DLL (ready to use with various interfaces)
- improving 3D geometry viewers

# More Examples & Acknowledgements

- Real-time computation:
  - Superconducting Wiggler (coils)
  - Simple Hybrid Wiggler
  - Simple Quadrupole Magnet


- Special thanks to:
  - ESRF ID Group
  - Workshop Organizers
  - All Radia Users