Planar Hall effect in the Vertical Hall Sensor

Part II - Revised interpretation

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Summary

Due to the symmetrical structure of the Vertical Hall Sensor (VHS) there existed reasonable doubts, whether a magnetic field in the VHS plane will at all generate a "planar" Hall voltage. Therefore, the first investigations in the above matter, carried out a few years ago [3], were mainly an attempt to find an answer to the mentioned question, meanwhile the resulting equation (based on a mathematical model for the current distribution within the VHS) did not pretend to be numerically exact ([3], Art.. 6).

Meanwhile, detailed measurements of the VHS "planar" Hall voltage, performed recently ([4], [5]), brought up some differences between the experimental and theoretical results. Although the main discrepancy concerned a $\pi/2$-shift between the sinusoidal dependencies of the "planar" Hall voltage, the numerical values of the amplitudes of the latter one did not agree to well either.

Because there were no doubts about the correctness of the measurements, it became obviously necessary to check the previous theoretical solution for the "planar" Hall voltage. During the examination it was discovered - apart from some minor miscalculations and careless slips - an error in the formula for the angle between the curved current path and the magnetic field direction, which was responsible for the above mentioned discrepancy. Therefore, the aim of the present revised interpretation was to show that it is possible to achieve even with simple mathematical models theoretical solution for the strength of the "planar" Hall voltage, being now fully compatible with the experimental results.

Finally, it was proved that a combination of 2 VHS's, facing the normal magnetic field in opposite directions and whose Hall electrodes were connected to a differential amplifier, will show in an inhomogenous magnetic field at all times no "planar" Hall voltage at the amplifier output.

1. Introduction

The Vertical Hall Sensor (VHS) for precise magnetic field measurements represent a relatively new development (SENTRON AG, Zug, Switzerland). Since the early 90^{th}, when the VHS was introduced on the market, quite a few research and university laboratories have made use of its various types and investigations of their properties were published at different occasions ([1] - [5]).

One of the most important properties of the Hall generators (HG) is the so called planar Hall effect *), caused by a magnetic field component in the HG plane. This effect produces a potential difference between the Hall electrodes ("planar" Hall voltage),which appears as an error at the measurement of a transversal field component. Therefore, due to the importance of the planar effect at measurements of inhomogenous magnetic fields, the theoretical investigations of the planar Hall effect as well as of the "planar" Hall voltage magnitude had the first priority [3]; the calculations were then slightly later followed by corresponding experiments ([4], [5]) .

As already mentioned in [3], due to the symmetric current distribution pattern within the VHS at no magnetic field one could easily conclude that such a HG type will not produce any "planar" Hall voltage. However, detailed investigations ([3] - [5]) proved the existence of a planar Hall effect in a VHS, though they have also brought up some differences between the theoretical solution and the experimental results. Therefore, the aim of the following calculations will be to detect possible errors in the theory and to present results compatible with the measurement data.

In addition, investigations of the influence of the planar Hall effect on the Hall voltage output of the "high-accuracy" and "very high-accuracy" SENTRON probes showed no errors due to the "planar" Hall voltage. This is due to a pair of VHS's in each of the probes, mounted in such a way that they generate in a transversal magnetic field Hall voltages of opposite signs. Because the voltage outputs of the VHS's are connected to a differential amplifier, the Hall voltage at the probe output will be doubled, but the "planar" Hall voltages canceled. Therefore, it is possible to accomplish with the above mentioned probes 2- and 3-dimensional magnetic field measurements free of any "planar" Hall voltage errors.

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*) Because the planar Hall effect is a magnetoresistive effect [6], the voltage produced by this effect is not a Hall voltage. Therefore, the word "planar" in connection with the Hall voltage will appear in this report within quotation marks.

- 2 -

2. Link to previous theoretical investigations

In order to establish the connection with the previous investigation of the planar Hall effect in the VHS's [3], the basic equation for the galvanomagnetic phenomena as well as the current distribution model in the VHS shall be reproduced here.

2.1 Basic equation for the "planar" Hall voltage

Next to the magnetoresistance and Hall effect, a third galvanomagnetic effect was investigated theoretically in the early 50 th. This effect occurs, when a current carrying Hall generator is placed in a magnetic field with a component in the HG plane. Because of the similarity to the (normal) Hall effect, it was described as the "planar Hall effect" ([6] ; [B1] , pp. 25 , 169) .

From the general electric field equation (vectorial notation) for a semiconductive HG [8]

$$\mathbf{E} = \rho_o \cdot \mathbf{J} + R_H \cdot [\mathbf{B} \times \mathbf{J}] - M \cdot \rho_o \cdot [\mathbf{B} \times [\mathbf{B} \times \mathbf{J}]] \qquad (1)$$

with
- B - Magnetic induction
- E - Electric field
- J - Current density
- R_H - Hall coefficient
- ρ_o - Resistivity at $B = 0$
- M - Transversal coefficient of resistivity

one can deduct the equation for the potential difference between the Hall electrodes

$$V = K_1 \cdot B_n \cdot I + K_2 \cdot (B_p)^2 \cdot \sin(2\psi) \cdot I = U_H + U_p \qquad (2)$$

with
- K_1 - constant
- K_2 - constant
- B_n - Transversal (normal) induction component
- B_p - Planar induction component
- I - Hall generator control (supply) current
- ψ - Angle (B_p, I)

The first term in the equation (2) is the Hall voltage U_H *) and the second term describes the voltage part due to the planar Hall effect, which was investigated in detail in connection with the conventional (horizontal) HG's already many years ago ([7] - [10]) . In what follows, we shall be dealing only with this voltage part called "planar" Hall voltage (cf. also footnote on p. 1).

*) For reason of simplicity, the geometrical function in the Hall voltage term is considered to be equal to 1 over the full B-range .

2.2 Current distribution pattern in the VHS - physical facts

One of the main differences between a conventional HG and a VHS is the location of the current and Hall electrodes. While each side of the Hall plate of a conventional HG carries one electrode, on the VHS all of them are located on one side ([1], [2]). This has besides a beneficial influence on the linearity of the $U_H = f(B_n)$ of the VHS also (compared to the conventional HG) a completely different current and equipotential line patterns within the sensor (Fig. 1).

The fact that the supply current in a HG flows (even under the influence of a transversal field component) always in one direction, has made the mathematical treatment of the planar Hall effect and the calculation of the "planar" Hall voltage more or less straightforward ([7] - [10]). However, the VHS supply current, which is entering the middle electrode, flows further - divided into two halves - in opposite directions to the outer left and right current electrode (Fig. 1).

Due to the above and with no transversal field component, the current lines will show a symmetrical pattern (the virtual vertical line of symmetry goes through the middle current electrode). On the other hand, the influence of a transversal field component will destroy this symmetry, pushing the current lines either to the left or to the right,depending on the field direction influencing the VHS. Therefore, each of the VHS halves must be treated mathematically in a separate way ([3], Art. 4.3), which will not be a matter of further discussion in this report.

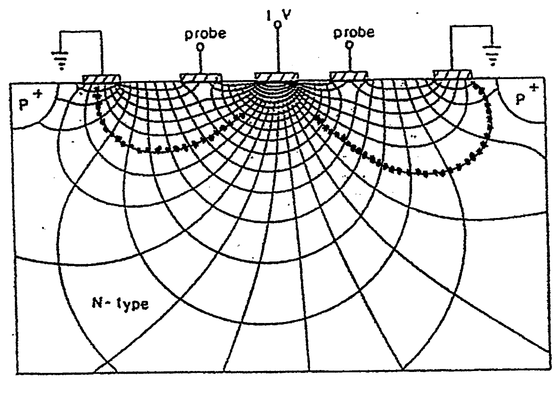

Fig. 1

Cross-section of a VHS with current and equipotential lines at a transversal magnetic induction of approx. 1 T (reproduced from [B2] by courtesy of R. S. Popovic)

2.3 Current distribution pattern in the VHS - symmetric case

Though the asymmetric pattern of the current lines in Fig. 1 was calculated for a transversal induction strength of the order of 1 T, it shall not be too difficult to follow the pattern back to the symmetric case ($B_n = 0$). In the following one can in the 1st approximation anticipate (cf. Art. 3.3) that each of the distributed current halves is concentrated in a "current filament", exiting in the VHS from the center of the middle current electrode and entering the outer left and right current electrodes (cf. [3], Art. 3.1). By doing this, one can easily figure out that the current filaments describe one half of a curve, known as a lemniscate (see dotted curve on the left side of Fig. 1). Using the same one as a basis for further calculations, one can now estimate the effect of a planar magnetic induction on the behavior of the VHS (cf. Art. 3.2).

3. Planar" Hall voltage of a single VHS

3.1 Model for the calculation procedure (introduction)

As already stated in Art. 2.3 above, the current distribution in each of the VHS may be represented by two current filaments. Dividing the filaments into infinitesimally small increments one can observe that they are steadily changing their direction along the filament. Therefore, one has at first to determine the contribution of each of the current filament increments to the "planar" Hall voltage (cf. definition of the angle ψ). Afterwards, the total "planar" Hall voltage for each half of the VHS follows by integration over the full length of the corresponding filament and the sum of both „planar" Hall voltages will then deliver the „planar" Hall voltage of the VHS.

As already shown in ([3] - [5]) , there are no doubts about the existence of the VHS "planar" Hall voltage. However, the determination of its magnitude (cf. [3], Art. 4.2) incorporated - apart from some calculation mistakes and careless slips) - a logical error, which lead to incorrect results (cf. explanations in Art. 3.2 below). Therefore, the magnitude determination of the "planar" Hall voltage must be revised, taking now also into the consideration (besides the elimination of all of the above mentioned errors) the fact of a vertical current density distribution in the VHS (cf. Art. 3.3).

3.2 Revised calculation procedure

In order to understand where the logical error has been made, it is necessary to repeat the calculation procedure from the beginning (cf. [3], Art. 4.2) .

Before the contribution of a current increment to the "planar" Hall voltage for each half of the VHS will be calculated, one must at first determine the length of a quarter of the lemniscate. This can be made easily in the polar coordinate system, where the length of a curve within the integration boundaries φ_1 and φ_2 is given by

$$s = \int_{\varphi_1}^{\varphi_2} [r^2(\varphi) + r'^2(\varphi)]^{1/2} \cdot d\varphi \qquad (3)$$

with $$r'^2(\varphi) = dr/d\varphi \qquad (3a)$$

Based on (3) one can now determine from the lemniscate equation $r(\varphi) = a_L \cdot \cos^{1/2}(2\varphi)$ **[$a_L = r(0)$] (Fig. 2) the length of an element of the current filament as** $ds = a_L \cdot \cos^{-1/2}(2\varphi) \cdot d\varphi$ **. Within integration boundaries [0, $\pi/4$] it follows for the length of a quarter of the lemniscate *)**

$$s_L = a_L \cdot \int_{\varphi=0}^{\pi/4} \cos^{-1/2}(2\varphi) \cdot d\varphi = (a_L/\sqrt{2}) \cdot \int_{\theta=0}^{\pi/2} [1-(1/2)\cdot \sin^2\theta]^{-1/2} \cdot d\theta = \qquad (3b)$$

$$= (a_L/\sqrt{2}) \cdot K(k) = a_L \cdot 1{,}31105$$

with $k^2 = 1/2$ and $K(k)$ as the complete elliptic integral of the first kind ([B3], p. 171, item 773.1.) .

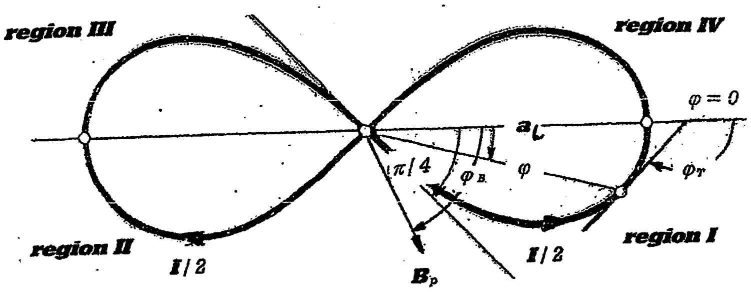

Fig. 2

Lemniscate function with B_p, φ, φ_B and φ_T

In the next step one has to determine the angle φ_T between the tangent to the increment of the current filament, the latter one having coordinates [$r(\varphi)$, φ], and the polar axis ($\varphi = 0$) **). According to the general equation

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*) **The result for s_L in [3] was due to a miscalculation approx.** 20% **to high.**

) **This angle will enter later into the equation for the "planar" Hall voltage (cf. $\sin(2\psi)$ **in (2)).**

$$tg\ \varphi_T = \frac{r'(\varphi)\cdot\sin\varphi + r(\varphi)\cdot\cos\varphi}{r'(\varphi)\cdot\cos\varphi - r(\varphi)\cdot\sin\varphi} \qquad (4)$$

one will obtain after introducing into (4) the formula for the lemniscate and its first derivative

$$\varphi_T = 3\varphi + \pi/2 \qquad (4a)$$

At this point was made the logical mistake in [3], using (4a) for further calculations and not considering that the current and the positive trigonometric direction *) on the right half of the VHS (region I on Fig. 2) are opposing. Taking this fact into account **) one will finally get

$$\varphi'_T = 3\varphi + \pi/2 - \pi = 3\varphi - \pi/2 \qquad (4b)$$

The angle between the directions of the planar field (induction) and the increment of the current filament is then given by

$$\psi = \varphi_B - \varphi'_T = \varphi_B - 3\varphi + \pi/2 \qquad (4c)$$

Based on (2) and the above result one can now calculate the "planar" Hall voltage U_p over the length of one quarter of the lemniscate (cf. right half of Fig. 2)

$$(U_p)_I = K_2\cdot(B_p)^2\cdot(I/2)\cdot(1/s_L)\cdot\int_{\varphi=0}^{\pi/4}\sin[2(\varphi_B - \varphi'_T)]\cdot ds = \qquad (5)$$

$$= K_2\cdot(B_p)^2\cdot(I/2)\cdot(a_L/2)^{-1}\cdot(1/2{,}62209)\,\cdot$$

$$\cdot\int_{\varphi=0}^{\pi/4}\sin[2(\varphi_B - 3\varphi + \pi/2)]\cdot(a_L/2)\cdot\cos^{-1/2}(2\varphi)\cdot d(2\varphi) =$$

$$= K_2\cdot(B_p)^2\cdot(I/2)\cdot J_1$$

Instead of $\sin[2(\varphi_B - 3\varphi + \pi/2)]$ **one can write** $-\sin[2(\varphi_B - 3\varphi)]$ **and define the first integral J_1 as follows**

***) The virtual y-axis of the chosen coordinate system is showing downwards and the positive angle is increasing clockwise, starting from the polar axis ($\varphi = 0$).**

****) In [3] it was taking into account instead that the current and the <u>integration</u> in the right half of the VHS have opposite directions ("–" sign in front of the integral), which in the last consequence lead to an incorrect result.**

$$\mathfrak{J}_1 = -(1/2{,}62209) \cdot \int_{\varphi=0}^{\pi/4} \sin[\,2\,(\varphi_B - 3\varphi)\,] \cdot \cos^{-1/2}(2\varphi) \cdot d(2\varphi) \qquad (5a)$$

It is possible to define the integral $\mathfrak{J}_2$ for the left half of the VHS (region II on Fig. 2) in a similar way, but in this case the current and the clockwise increasing φ are showing in the same direction. Because the expression (4a) is now valid one obtains with integration within the boundaries [$3\pi/4$, π])

$$(U_p)_{II} = K_2 \cdot (B_p)^2 \cdot (I/2) \cdot (1/s_L) \cdot \int_{\varphi=3\pi/4}^{\pi} \sin[\,2\,(\varphi_B - \varphi_T)\,] \cdot ds = \qquad (6)$$

$$= K_2 \cdot (B_p)^2 \cdot (I/2) \cdot (a_L/2)^{-1} \cdot (1/2{,}62209) \cdot$$

$$\cdot \int_{\varphi=3\pi/4}^{\pi} \sin[\,2\,(\varphi_B - 3\,\varphi - \pi/2)\,] \cdot (a_L/2) \cdot \cos^{-1/2}(2\varphi) \cdot d(2\varphi)$$

The substitution $\varphi = \pi - \phi$ leads to integration boundaries [0 , $\pi/4$] and, replacing formally at the end of the calculation ϕ with φ , one will finally get for the integral

$$\mathfrak{J}_2 = -(1/2{,}62209) \cdot \int_{\varphi=0}^{\pi/4} \sin[\,2\,(\varphi_B + 3\varphi)\,] \cdot \cos^{-1/2}(2\varphi) \cdot d(2\varphi) \qquad (6a)$$

For the sum of $\mathfrak{J}_1$ and $\mathfrak{J}_2$ follows after substitution $x = 2\varphi$ to

$$\sum_{i=1}^{2} \mathfrak{J}_i = -(2/2{,}62209) \cdot \sin 2\varphi_B \cdot \int_{x=0}^{\pi/2} (1 - 4 \cdot \sin^2 x) \cdot \cos^{1/2} x \cdot dx = \qquad (7)$$

$$= -(2/2{,}62209) \cdot \sin 2\varphi_B \cdot (I_1 + I_2)$$

The solution of I_1 consists of a combination of the complete elliptic integrals of the first and second kind ([B4],p.181,item 2.618/11)

$$I_1 = \int_{x=0}^{\pi/2} \cos^{1/2} x \cdot dx = 1{,}1981 \qquad (7a)$$

while the solution of I_2 can be obtained using the general equation ([B3], p. 100, item 450.9)

$$\int \sin^n(x) \cdot \cos^m(x) \cdot dx$$

with $n = 2$ and $m = ½$, leading to

$$I_2 = -4 \cdot \{ [-(2/5) \cdot \sin(x) \cdot \cos^{3/2}(x)]_{x=0}^{\pi/2} + (2/5) \cdot \int_{x=0}^{\pi/2} \cos^{1/2} x \cdot dx \} = \quad (7b)$$

$$= -4 \cdot (2/5) \cdot 1{,}1981$$

and with $$\sum_{i=1}^{2} I_i = 1{,}1981 - 4 \cdot (2/5) \cdot 1{,}1981 = -0{,}71886 \quad (7c)$$

finally to $$\sum_{i=1}^{2} J_i = -(2/2{,}62209) \cdot \sin 2\varphi_B \cdot (-0{,}71886) = 0{,}5483 \cdot \sin 2\varphi_B \quad (7d)$$

The total "planar" Hall voltage for a VHS follows then, using the result in (7), by addition of the partial "planar" Hall voltages $(U_p)_I$ and $(U_p)_{II}$ according to (5) and (6)

$$(U_p)_{VHS} = K_2 \cdot (B_p)^2 \cdot (I/2) \cdot 0{,}5483 \cdot \sin 2\varphi_B \quad (8)$$

The solution (8) can be used now to determine the magnitude of the VHS "planar" Hall voltage. Inserting the <u>absolute</u> value of K_2 according to [10]

$$K_2 = -\rho_o \cdot M / 2d \quad (8a)$$

into (8), with d as the depth of the VHS (corresponding to the thickness of the conventional HG), one will get expression for the maximum value (amplitude) of the "planar" Hall voltage

$$[(U_p)_{VHS}]_{max} = 0{,}5483 \cdot [\rho_o \cdot M / 2d] \cdot (B_p)^2 \cdot (I/2) \quad (9)$$

With the data of the Si-VHS I04S04 ([4], p. 56; cf. Table I.), used for the measurements of the "planar" Hall voltage ([4], p. 82),one can

$n \doteq 2 \cdot 10^{14} cm^{-3}$	**([4], p. 25)**
$\rho_o = 30\ Vcm/A$	**([B2], p. 41, Fig. 2.10)**
$M = 1{,}55\ \%/T^2$	**([B2], p. 138, Fig. 3.33)**
$d = 108\ \mu m$	**([4], p. 55)**
$a_L = 116\ \mu m$	**([4], p. 56, Tab. 3.1; cf. (3b))**
$h_c \doteq 500\ \mu m$	***)**

Table I.

VHS characteristic data
(reproduced from [4] and [B2] by courtesy of C. Schott and R.S. Popovic, respectively)

***) The use of h_c (chip thickness) will be explained in Art. 3.3 of this report.**

calculate the reduced amplitude of the "planar" Hall voltage for the total sensor current $I = 1\ mA$

$$[(U_p)_{VHS}]_{max} / (B_p)^2 = 0{,}5483 \cdot [\rho_o \cdot M / 2d] \cdot (I/2) = 5{,}90184\ mV/T^2 \quad \textbf{(10a)}$$

Now it is possible to compare the "planar" Hall voltages according to (10a) with the measurement results ([4], [11]). However, one can not use the raw data directly, because they contain - in addition to the "planar" Hall voltages - also the voltage errors due to the positioning displacement(s) of the VHS in the measuring set up. Therefore, the raw data must be at first treated in a way, which allows to separate both kinds of voltages. This can be made using Fourier analysis, because the voltage due to the positioning errors of the VHS in the measuring set up varies with a period 2π ***, while the "planar" Hall voltage is changing its value with a period*** π***. That way one can finally determine the actual strength of the „planar" Hall voltage, hidden in the measured (raw) data.***

However, before comparing the theoretical with the "clean" measured data it is necessary to analyze the latter one (cf. Table II., last column). Based on the values between $0{,}5\ T$ ***and*** $2{,}0\ T$ ***one can easily find out that the measured data do not follow the*** $(B_p)^2$***-law rather then the*** $(B_p)^n$***-law (*** $n \doteq 1{,}47$ ***) *). Using the abbreviation*** $\hat{U}_p$ ***for*** $[(U_p)_{VHS}]_{max}$ ***one can now calculate with the help of (10a) the theoretical "planar" Hall voltages for both laws and compare them with the measured data (cf. Table II.).***

Induction	***Theory***		***Measurement***
B_p [T]	$\hat{U}_p$ ($n \doteq 2$) [mV]	$\hat{U}_p$ ($n \doteq 1{,}47$) [mV]	$\hat{U}_p$ ($n \doteq 1{,}47$) [mV]
0,5	1,475	2,131	0,519
1,0	5,902	5,902	1,460
1,5	13,279	10,711	2,608
2,0	23,607	16,349	4,023

Table II.

Theoretical and measurement results of the "planar" Hall voltage

****) Many measurements of the magnetoresistance effect (being also responsible for the planar Hall effect), performed during the past decades, have shown that the*** B^2***-law is valid only for low*** B***-values. Besides, the validity of the*** B^2***-law depends for a particular semiconductor also upon the donor concentration.***

In order to compare the course of the „planar" Hall voltage according to (8) and (8a) with that one in [4] (p. 85, Fig. 3.38) one has to make a coordinate transformation. This is necessary, because in this report the sensor is considered to be in a steady position and the planar field is rotating with an angle φ_B . In the experiment is on the contrary the field remaining in a steady position while the sensor is rotating. Using the denotation φ_R for the sensor rotating angle it is possible to switch from the experimental coordinate system to that one in this paper by the relation $\varphi_B = = \varphi_R + \pi/2$.

The next comparison between the theoretical and measurement results for $n \doteq 1{,}47$ in Table II. reveals a difference in the magnitude of the "planar" Hall voltages by a factor of approx. 4. Because only a very small portion of this difference can be attributed to the uncertainty of the VHS characteristic data (cf. Table I.), one has to search now for the major reason of the above mentioned discrepancy (cf. Art. 3.3).

3.3 Equivalent current distribution in the VHS

Analyzing more in detail the current distribution (current lines) within the VHS (cf. Fig. 1) one can figure out that the anticipation of the current $I/2$, "carried" in each half of the VHS by the corresponding lemniscate "current filament", is somehow a crude one (cf. Art. 2.3). Therefore, it will be necessary to find an appropriate equivalent for the - rather complicated - current distribution in the VHS (cf. Fig. 1) and attribute to the lemniscate the correct current strength.

Because the VHS input supply voltage is the same for all current lines, the current strength at the individual lines will more or less rapidly decrease with the distance from the chip surface. This is due to the fact that the length of the lines becomes longer with their distance from the chip surface and - consequently - therefore the resistance of each current "tube" larger.

Keeping the above in mind, one can now establish a general equation for the length of a current line as a function of the distance y from the chip surface. In the 1^{st} approximation one may write

$$l_y = [\,(a_L)^2 + (y/k)^2\,]^{1/2} \qquad (11)$$

The constant k in the above equation can be determined knowing the length of one quarter of the lemniscate ($s_L = a_L \cdot 1{,}31105$; (3b)) and the distance from its farthest point in y-direction from the chip surface ($y_L = a_L / (2\sqrt{2})$; [B5], p. 90) . With these parameters it follows for the constant

$$k = 0{,}417 \qquad (12)$$

In the next step one has to determine the equivalent length l_{eq} of <u>all</u> current lines within the chip (integral of l_y in y-direction over the full chip thickness h_L , reduced to the chip thickness). The (virtual) equivalent length l_{eq} is therefore of particular interest, because it is <u>the</u> filament, which is "carrying" the current $I/2$ of one half of the sensor.

After few substitutions in the simple integral and with the data from Table I. one obtains for the equivalent length

$$l_{eq} = (1/h_L) \cdot \int_{y=0}^{h} [a_L^2 + (y/k)^2]^{1/2} \cdot dy = a_L \cdot 5{,}339 \qquad (13)$$

The equivalent length of all current lines reduced to the length of the lemniscate "current filament" follows then from (13) and (3b)

$$l_{eq} / s_L = 5{,}339 / 1{,}31105 = 4{,}0723 \qquad (14)$$

As already stated above, the Hall supply voltage is the same for all current lines in each half of the VHS. Therefore, it is possible to establish the relation between $I/2$ and l_{eq} on one side as well as between i_L and s_L on the other side. Taking also into account that the cross-section of the current path is approximately proportional to the square of the length of the current line, one can write

$$I/2 \sim l_{eq} \qquad \text{and} \qquad i_L \sim s_L$$

or finally

$$i_L = I/2 \cdot s_L / l_{eq} = I/2 \cdot 0{,}24556 \qquad (15)$$

The equation (15) shows that the actual strength of the current, "carried" by the corresponding part of the lemniscate, is <u>not</u> $I/2$ - as anticipated in Art. 2.3. Instead, the strength of the current through that part of the lemniscate is by a factor of 0,24556 <u>smaller,</u> leading to

$$[(U_p)_{VHS}]_{max} / (B_p)^n = 0{,}5483 \cdot [\rho_o \cdot M / 2d] \cdot (I/2) = 1{,}449 \, mV/T^2 \qquad (10b)$$

with n equals either to 2 or 1,47 (cf. explanations in Art. 3.2).

Based on this fact, one can now compare the measurement results (cf. Table II, last column) with the theoretical values for $n \doteq 1{,}47$ (cf. Table II, 3rd column), being now revised according to (15) (see Table III and conclusions in Art. 5.).

Induction	Theory (equivalent current distribution)	Measurement
B_p [T]	$\hat{U}_p$ (n ≐ 1,47) [mV]	$\hat{U}_p$ (n ≐ 1,47) [mV]
0,5	0,523	0,519
1,0	1,449	1,460
1,5	2,629	2,608
2,0	4,013	4,023

Table III.

Revised theoretical results and measurement data of the "planar" Hall voltage

4. "Planar" Hall voltage of high-accuracy VHS probes

The special SENTRON "high-accuracy" and "very high-accuracy" probes contain 2 single VHS's with possibly equal characteristics. The sensors are located within the probe in such a way that the magnetic field is entering them from different sides, generating Hall voltages of opposite signs. Connecting the Hall outputs of the sensors to a differential amplifier (Fig. 3), the same one will then deliver at its output twice the single Hall voltage. However, the offset voltages of the sensors, which are independent of the field direction, will be practically fully canceled ([1], [2]).

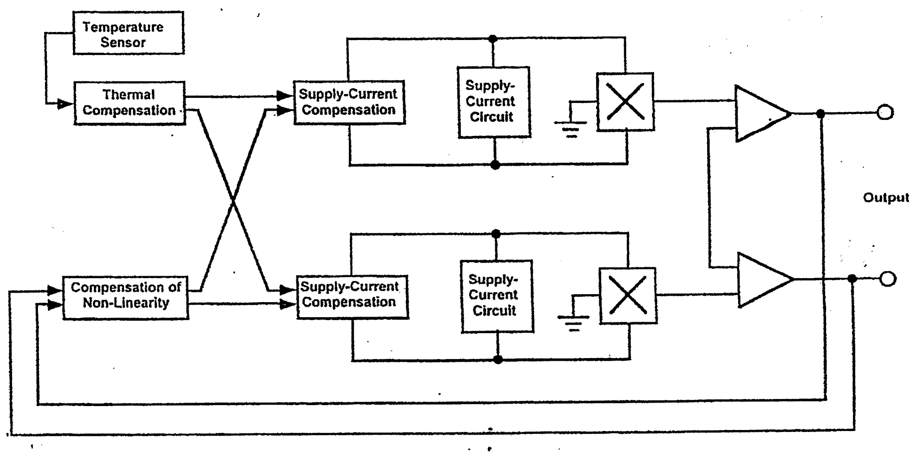

Fig. 3

Simplified block diagram of a "high-accuracy" VHS probe (reproduced from [1] by courtesy of R. S. Popovic)

Though the above described combination of 2 VHS's shows definite advantages compared to a single VHS probe, the question, whether in an inhomogenous magnetic field the probe output will be free of the "planar" Hall voltage or not, still remains open.

In order to find an answer to the crucial question it is necessary to start with Fig. 1. Following the explanations in Art. 2.2, 3rd par., one shall disregard the fact that the current and potential lines correspond to the influence of a transversal magnetic field. With other words, one shall consider the picture being symmetric around the virtual vertical axis going through the middle current electrode and the main current lines have the form of a lemniscate (see dotted line on the left side of Fig. 1).

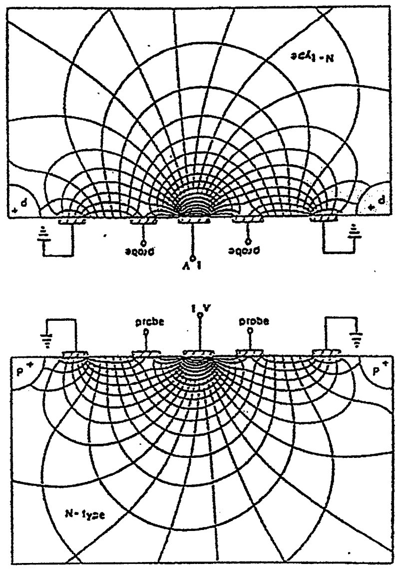

Fig. 4

Current and equipotential lines of 2 inverted sensors in a VHS probe, influenced by a transversal magnetic induction

In the following step one shall imagine a second VHS according to Fig. 1 and precedent remarks, but rotated upwards (out of the paper plane) by 180° around a virtual axis located slightly above the original (lower) VHS (see Fig. 4). Anticipating both VHS's being exactly in the same plane, it is possible to repeat in the same way the calculations made in Art. 3.2 . For the region III one shall then obtain

$$(U_p)_{III} = K_2 \cdot (B_p)^2 \cdot (I/2) \cdot (1/s_L) \cdot \int_{\varphi=\pi}^{5\pi/4} \sin[2(\varphi_B - \varphi'_T)] \cdot ds$$

and with a simple transformation and substitution

$$(U_p)_{III} = K_2 \cdot (B_p)^2 \cdot (I/2) \cdot$$

$$\cdot (-1/2{,}62209) \cdot \int_{\varphi=0}^{\pi/4} \sin[2(\varphi_B - 3\varphi)] \cdot \cos^{-1/2}(2\varphi) \cdot d(2\varphi) =$$

$$= K_2 \cdot (B_p)^2 \cdot (I/2) \cdot J_1$$

which is identical with (5) and (5a).

After similar procedure one can prove that for the region IV (with integration boundaries [π , $5\pi/4$]) follows the same result as for the region II , i.e. (6) and (6a).

Because the polarity of the Hall electrodes of the upper VHS remains unchanged with respect to those of the lower sensor and both "planar" Hall voltages have according to the above calculations the same sign, the output of the differential amplifier will be therefore free of any "planar" Hall voltages.

5. Conclusions

From the expression (10b) for the VHS "planar" Hall voltage and Table III. follows that - contrary to the statements in ([4], p. 80/81) and ([5], p.438) - it is even with relatively simple mathematical models obviously possible to estimate the strength of the "planar" Hall voltage with a large degree of confidence. However, the excellent agreement between the theoretical and experimental data in Table III. is rather accidental than intentional.

In addition, using two VHS's, located within a probe and connected as described in Art. 4., it is definitely possible to avoid errors due to the "planar" Hall voltages at measurements of inhomogenous magnetic fields.

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